Higher Maths 2005

Paper 1 Solutions

1.
$$m = \tan 60°$$
$$m = \sqrt{3}$$

$$y - b = m(x - a)$$
$$y - 0 = \sqrt{3}\,(x - (-2))$$
$$y = \sqrt{3}\,x + 2\sqrt{3}$$

2. (a) Congruent circles $\Rightarrow$ radii equal

P = midpoint of AB

From equations; A(–3, –2) and B(3,6)

$\Rightarrow$ P(0,2)

(b)
$$\text{AB} = \sqrt{(3-(-3)^2 + (6-(-2))^2}$$
$$= \sqrt{(6)^2 + (8)^2}$$
$$= \sqrt{100}$$
$$= 10 \text{ units}$$

3. (a)
$$\overrightarrow{DF} = \frac{2}{3}\overrightarrow{DB}$$
$$\overrightarrow{DF} = \frac{2}{3}\begin{pmatrix} 6 \\ 3 \\ -9 \end{pmatrix}$$
$$\overrightarrow{DF} = \begin{pmatrix} 4 \\ 2 \\ -6 \end{pmatrix}$$

$\Rightarrow$ F(10,5,3)

(b)
$$\overrightarrow{AF} = \overrightarrow{AD} + \overrightarrow{DF}$$
$$\overrightarrow{AF} = \begin{pmatrix} -6 \\ 3 \\ 9 \end{pmatrix} + \begin{pmatrix} 4 \\ 2 \\ -6 \end{pmatrix}$$
$$\overrightarrow{AF} = \begin{pmatrix} -2 \\ 5 \\ 3 \end{pmatrix}$$

4. (a)
$$h(x) = g(f(x))$$
$$= g(3x - 1)$$
$$= (3x - 1)^2 + 7$$

(b) (i) Minimum turning point at (⅓,7)

(ii) Range: $y \geq 7$

5.
$$\frac{d}{dx}(1 + 2\sin x)^4 = 4(1 + 2\sin x)^3 . 2\cos x$$
$$= 8\cos x(1 + 2\sin x)^3$$

6. (a) $u_{n+1} = ku_n + 5$

$L = kL + 5$

$4 = 4k + 5$

$-1 = 4k$

$-\frac{1}{4} = k$

(b) (i) $u_{n+1} = mu_n + 5$ $\quad u_o = 3$

$u_1 = 3m + 5$

$u_2 = m(3m + 5) + 5$

$u_2 = 3m^2 + 5m + 5$

(ii) $3m^2 + 5m + 5 = 7$

$3m^2 + 5m - 2 = 0$

$(3m - 1)(m + 2) = 0$

$\Rightarrow m = \frac{1}{3}$ or $m = -2$

$m = -2$ produces a sequence with no limit

as $-1 < m < 1$ for a limit to exist

7. (a) $a = 4$, $b = 5$

(b) Domain: $x > 4$

8. (a)

3	2	–7	0	9
		6	–3	–9
	2	–1	–3	0

$\Rightarrow (x - 3)$ is a factor

Quotient: $2x^2 - x - 3$

$\Rightarrow f(x) = (x - 3)(2x^2 - x - 3)$

$= (x - 3)(x + 1)(2x - 3)$

(b) $f(x)$ crosses x-axis when $y = 0$

$\Rightarrow x = -1, \frac{3}{2}, 3$

$\Rightarrow (-1,0) \quad (^3/_2,0) \quad (3,0)$

$f(x)$ crosses y-axis when $x = 0$

$\Rightarrow y = 9$

$\Rightarrow (0,9)$

8. (c) $f'(x) = 6x^2 - 14x$

For stationary points, $f'(x) = 0$

$\Rightarrow$ $6x^2 - 14x = 0$

$2x(3x - 7) = 0$

$\Rightarrow$ $x = 0$ or $x = \frac{7}{3}$

when $x = 0$, y = 9

when $x = \frac{7}{3}$, $y = 2\left(\frac{7}{3}\right)^3 - 7\left(\frac{7}{3}\right) + 9$

$= \frac{686}{27} - \frac{343}{9} + 9$

$= \frac{686}{27} - \frac{1029}{27} + \frac{243}{27}$

$= \frac{-100}{27}$

x	$\rightarrow$	0	$\rightarrow$	$\frac{7}{3}$	$\rightarrow$
$f'(x)$	+	0	–	0	+
	/	–	\	_	/

$\Rightarrow$ Max TP at (0,9)

$\Rightarrow$ Min TP at $\left(\frac{7}{3}, \frac{-100}{27}\right)$

when $x = -2$, $y = 2(-2)^3 - 7(-2) + 9$

$= -16 - 28 + 9$

$= -35$

$(-2, -35)$

$y = 2(2)^3 - 7(2) + 9$

$= 16 - 28 + 9$

$= -3$

$(2, -3)$

$\Rightarrow$ greatest value of f in the interval $-2 \leq x \leq 2$ is 9 and least value of f is -35

9.

$\cos 2x = \frac{7}{25}$	$\cos 2x = \frac{7}{25}$
$2\cos^2 x - 1 = \frac{7}{25}$	$1 - 2\sin^2 x = \frac{7}{25}$
$2\cos^2 x = \frac{32}{25}$	$-2\sin^2 x = -\frac{18}{25}$
$\cos^2 x = \frac{16}{25}$	$\sin^2 x = \frac{9}{25}$
$\cos x = \pm\frac{4}{5}$	$\sin x = \pm\frac{3}{5}$

$\Rightarrow \cos x = \frac{4}{5}$ and $\sin x = \frac{3}{5}$ for $0 < x < \frac{\pi}{2}$ (only 1st quadrant solutions required)

10. (a)

$$\sin x - \sqrt{3}\cos x$$

$$k\sin(x-a) = k\sin x\cos a - k\cos x\sin a$$

$k\cos a = 1$

$k\sin a = \sqrt{3}$

$k = \sqrt{(1)^2 + (\sqrt{3})^2}$

$k = \sqrt{1+3}$

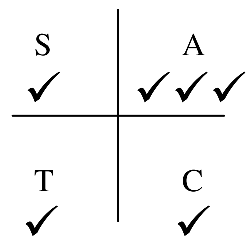
$k = \sqrt{4}$

$k = 2$

$\tan a = \frac{k\sin a}{k\cos a}$

$\tan a = \frac{\sqrt{3}}{1}$

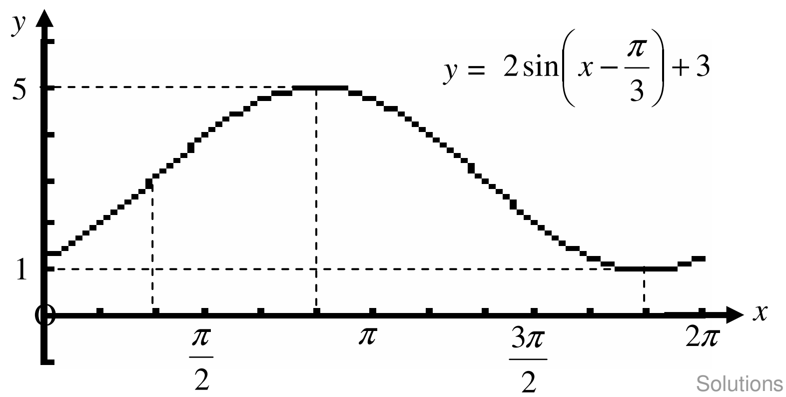
$a = \frac{\pi}{3}$



a is in 1st quadrant so $a = \frac{\pi}{3}$

$$\Rightarrow \sin x - \sqrt{3}\cos x = 2\sin\left(x - \frac{\pi}{3}\right)$$

(b)



11. (a) $(x - t)^2 + (y - 0)^2 = (2)^2$

$$(x - t)^2 + y^2 = 4$$

(b) $(x - t)^2 + (2x)^2 = 4$

$$x^2 - 2tx + t^2 + 4x^2 = 4$$

$$5x^2 - 2tx + t^2 - 4 = 0$$

If $y = 2x$ is tangent then $b^2 - 4ac = 0$

$$(-2t)^2 - 4(5)(t^2 - 4) = 0$$

$$4t^2 - 20t^2 + 80 = 0$$

$$80 - 16t^2 = 0$$

$$80 = 16t^2$$

$$5 = t^2$$

$$t = \pm\sqrt{5}$$

$\Rightarrow t = \sqrt{5}$ as t is on positive x-axis in diagram