

Higher Maths 2005

Paper 2 Solutions

$$\begin{aligned} 1. \quad & \int \frac{4x^3 - 1}{x^2} x \\ &= \int \left(\frac{4x^3}{x^2} - \frac{1}{x^2} \right) dx \\ &= \int (4x - x^{-2}) dx \\ &= \frac{4x^2}{2} - \frac{x^{-1}}{-1} + c \\ &= 2x^2 + \frac{1}{x} + c \end{aligned}$$

$$2. \quad (a) \sin(p + q) = \sin p \cos q + \cos p \sin q$$

$$= \frac{15}{17} \cdot \frac{4}{5} + \frac{8}{17} \cdot \frac{3}{5}$$

$$= \frac{60}{85} + \frac{24}{85}$$

$$= \frac{84}{85}$$

$$(b) \cos(p + q) = \cos p \cos q - \sin p \sin q$$

$$= \frac{8}{17} \cdot \frac{4}{5} - \frac{15}{17} \cdot \frac{3}{5}$$

$$= \frac{32}{85} - \frac{45}{85}$$

$$= -\frac{13}{85}$$

$$(c) \tan(p + q) = \frac{\sin(p + q)}{\cos(p + q)}$$

$$= \frac{\frac{84}{85}}{-\frac{13}{85}}$$

$$= \frac{84}{85} \times -\frac{85}{13}$$

$$= -\frac{84}{13}$$

$$3. \quad (a) \quad m_{AB} = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

$$= \frac{(4 - 0)}{(5 - 1)}$$

$$= \frac{4}{4}$$

$$= 1$$

$\Rightarrow m_{\text{chord}} = -1$ as $m_1 \cdot m_2 = -1$ for \perp lines

midpoint = (3,2)

$$y - b = m(x - a)$$

$$y - 2 = -1(x - 3)$$

$$y - 2 = -x + 3$$

$$x + y = 5$$

$$(b) \quad m_T = -\frac{1}{3}$$

$$\Rightarrow m_T = 3$$

$$y - b = m(x - a)$$

$$y - 0 = 3(x - 1)$$

$$y = 3x - 3$$

(c) Perpendicular bisector of AB (answer (a))

and radius CA will intersect at centre, C.

$$\Rightarrow x + (3x - 3) = 5$$

$$4x = 8$$

$$x = 2$$

$$\Rightarrow y = 3$$

$$\Rightarrow C(2,3)$$

$$r = \sqrt{(x_C - x_A)^2 + (y_C - y_A)^2}$$

$$r = \sqrt{(2 - 1)^2 + (3 - 0)^2}$$

$$r = \sqrt{(1)^2 + (3)^2}$$

$$r = \sqrt{10}$$

$$\Rightarrow \text{Equation of circle: } (x - 2)^2 + (y - 3)^2 = 10$$

$$4. \text{ (a) } \vec{TA} = \begin{pmatrix} -5 \\ 15 \\ 1 \end{pmatrix} \quad \vec{TB} = \begin{pmatrix} -40 \\ 15 \\ 2 \end{pmatrix}$$

$$\begin{aligned} \text{(b) } |\vec{TA}| &= \sqrt{(-5)^2 + (15)^2 + (1)^2} \\ &= \sqrt{251} \end{aligned}$$

$$\begin{aligned} |\vec{TB}| &= \sqrt{(-40)^2 + (15)^2 + (2)^2} \\ &= \sqrt{1829} \end{aligned}$$

$$\cos A = \frac{a \cdot b}{|a||b|}$$

$$\cos A = \frac{(-5)(-40) + (15)(15) + (1)(2)}{\sqrt{251} \cdot \sqrt{1829}}$$

$$\cos A = 0.631$$

$$A = 50.9^\circ$$

5. For points of intersection $y = y$

$$\Rightarrow 2x^2 - 9 = x^2$$

$$x^2 - 9 = 0$$

$$(x + 3)(x - 3) = 0$$

$$\Rightarrow x = -3, 3$$

$$A = \int_{-3}^3 (x^2 - (2x^2 - 9)) dx$$

$$= \int_{-3}^3 (9 - x^2) dx$$

$$= \left[9x - \frac{x^3}{3} \right]_{-3}^3$$

$$= \left(9(3) - \frac{(3)^3}{3} \right) - \left(9(-3) - \frac{(-3)^3}{3} \right)$$

$$= (27 - 9) - (-27 + 9)$$

$$= 18 - (-18)$$

$$= 36 \text{ units}^2$$

$$6. \quad y = \frac{24}{\sqrt{x}}$$

$$y = 24x^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = -12x^{-\frac{3}{2}}$$

$$\frac{dy}{dx} = \frac{-12}{\sqrt{(x)^3}}$$

when $x = 4$

$$y = \frac{24}{\sqrt{(4)}}$$

$$y = 12$$

$$\frac{dy}{dx} = \frac{-12}{\sqrt{(4)^3}}$$

$$\frac{dy}{dx} = -\frac{12}{8}$$

$$\frac{dy}{dx} = -\frac{3}{2}$$

$$\Rightarrow P(4,12) \quad m = -\frac{3}{2}$$

$$y - b = m(x - a)$$

$$y - 12 = -\frac{3}{2}(x - 4)$$

$$2y - 24 = -3x + 12$$

$$3x + 2y = 36$$

$$7. \quad \log_4(5 - x) - \log_4(3 - x) = 2$$

$$\log_4 \frac{(5 - x)}{(3 - x)} = 2$$

$$4^2 = \frac{(5 - x)}{(3 - x)}$$

$$16 = \frac{(5 - x)}{(3 - x)}$$

$$16(3 - x) = 5 - x$$

$$48 - 16x = 5 - x$$

$$43 = 15x$$

$$\frac{43}{15} = x$$

8. For points of intersection $y = y$

$$\Rightarrow k \sin 2x = \sin x$$

$$k \times 2 \sin x \cos x = \sin x$$

$$2k \sin x \cos x - \sin x = 0$$

$$\sin x (2k \cos x - 1) = 0$$

$$\Rightarrow \sin x = 0 \quad \text{or} \quad 2k \cos x - 1 = 0$$

$$x = 0, \pi, 2\pi \quad 2k \cos x = 1$$

$$\cos x = \frac{1}{2k}$$

\Rightarrow From diagram, $x = 0, \pi, 2\pi$ correspond to points O, B and D respectively.

Hence, at A and C, $\cos x = \frac{1}{2k}$.

9. (a) At launch, $t = 0$

$$\Rightarrow V = \text{£}252 \text{ million}$$

(b) $20 = 252e^{-0.06335t}$

$$\frac{20}{252} = e^{-0.06335t}$$

$$\log_e \left(\frac{20}{252} \right) = \log_e (e^{-0.06335t})$$

$$\log_e \left(\frac{20}{252} \right) = -0.06335t$$

$$t = \frac{\log_e \left(\frac{20}{252} \right)}{-0.06335}$$

$$t = 40 \text{ years}$$

$$\begin{aligned}
10. \quad & \mathbf{a \cdot (a + b + c)} \\
& = \mathbf{a \cdot a + a \cdot b + a \cdot c} \\
& = \mathbf{a^2 + |a| |b| \cos 90^\circ + |a| |c| \cos 60^\circ} \\
& = \mathbf{(3)^2 + 0 + (3)(3) \cos 60^\circ} \\
& = \mathbf{9 + 9 \cdot \frac{1}{2}} \\
& = \mathbf{13.5}
\end{aligned}$$

11. (a)

$$-1 \left| \begin{array}{ccc|c} 1 & p & p & 1 \\ & -1 & 1-p & -1 \\ \hline 1 & p-1 & 1 & 0 \end{array} \right.$$

$\Rightarrow x = -1$ is a solution

(b) Quotient: $x^2 - (p-1)x + 1$

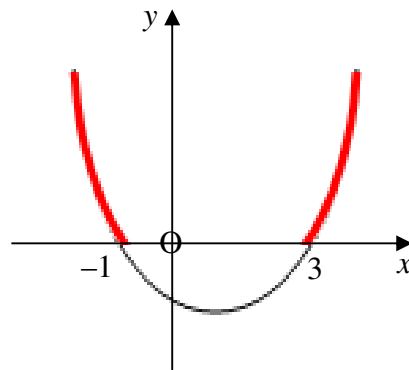
For real roots $b^2 - 4ac \geq 0$

$$(p-1)^2 - 4(1)(1) \geq 0$$

$$p^2 - 2p + 1 - 4 \geq 0$$

$$p^2 - 2p - 3 \geq 0$$

$$(p+1)(p-3) \geq 0$$



$\Rightarrow p \leq -1$ and $p \geq 3$ for all roots of the cubic function to be real.