Easter Differentiation

[SQA] 1. Find $f'(4)$ where $f(x) = \dfrac{x-1}{\sqrt{x}}$. **5**

2. If $y = 3x^{-2} + 2x^{\frac{3}{2}}$, $x > 0$, determine $\frac{dy}{dx}$.

A. $-6x^{-3} + \frac{4}{5}x^{\frac{5}{2}}$

B. $-3x^{-1} + 3x^{\frac{1}{2}}$

C. $-6x^{-3} + 3x^{\frac{1}{2}}$

D. $-3x^{-1} + \frac{4}{5}x^{\frac{5}{2}}$ **2**

[SQA] 3. Diagram 1 is an artist's impression of a new warehouse based on the architect's plans. The warehouse is in the shape of a cuboid and is supported by three identical parabolic girders spaced 30 metres apart.
With coordinate axes as shown in Diagram 2, the shape of each girder can be described by the equation $y = 9 - \frac{1}{4}x^2$.

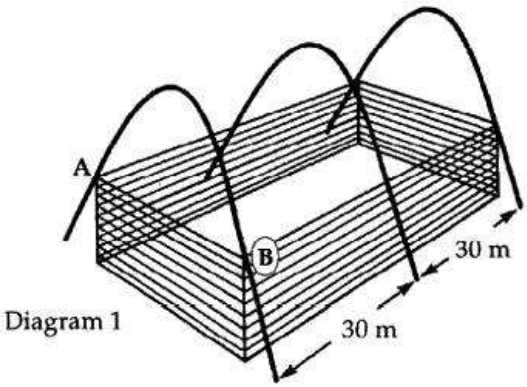


Diagram 1

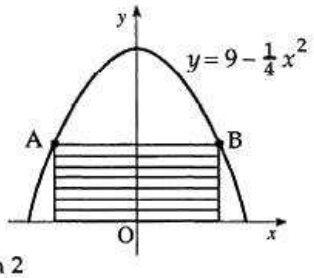


Diagram 2

(*a*) Given that AB is $2x$ metres long, show that the shaded area in Diagram 2 is $\left(18x - \frac{1}{2}x^2\right)$ square metres. **(2)**

(*b*) The architect wished to fit into the girders the cuboidal warehouse which had the maximum volume. Find the value of this maximum volume. **(6)**

4. If $f(x) = \frac{1}{\sqrt[5]{x}}$, $x \neq 0$, what is $f'(x)$?

   A. $-\frac{1}{5}x^{-\frac{6}{5}}$

   B. $-\frac{1}{5}x^{-\frac{4}{5}}$

   C. $-\frac{5}{2}x^{-\frac{7}{2}}$

   D. $-\frac{5}{2}x^{-\frac{3}{2}}$ **2**

5. A curve has equation $y = 5x^3 - 12x$.

   What is the gradient of the tangent at the point $(1, -7)$?

   A. -7

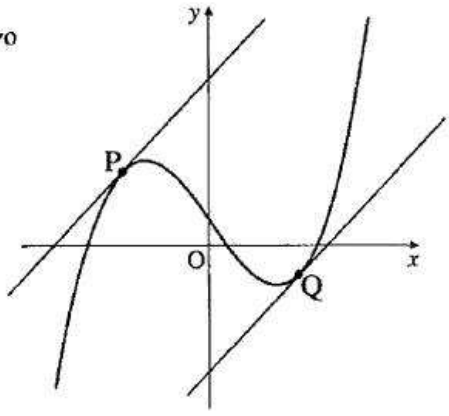
   B. -5

   C. 3

   D. 5 **2**

[SQA] 6. The point $P(-1, 7)$ lies on the curve with equation $y = 5x^2 + 2$. Find the equation of the tangent to the curve at P. **4**

[SQA] 7. The diagram shows a sketch of the graph of $y = x^3 - 9x + 4$ and two parallel tangents drawn at P and Q.



   (a) Find the equations of the tangents to the curve $y = x^3 - 9x + 4$ which have gradient 3. **6**

   (b) Show that the shortest distance between the tangents is $\frac{16\sqrt{10}}{5}$. **6**

[SQA] 8. A curve has equation $y = x - \frac{16}{\sqrt{x}}$, $x > 0$.

   Find the equation of the tangent at the point where $x = 4$. **6**

9. The volume of a sphere is given by the formula $V = \frac{4}{3}\pi r^3$. What is the rate of change of V with respect to r, at $r = 2$?

   A. $\frac{16\pi}{3}$

   B. $\frac{32\pi}{3}$

   C. 16π

   D. 32π **2**

[SQA] 10. For what values of x is the function $f(x) = \frac{1}{3}x^3 - 2x^2 - 5x - 4$ increasing? **5**

[SQA] 11. A curve has equation $y = 2x^3 + 3x^2 + 4x - 5$.

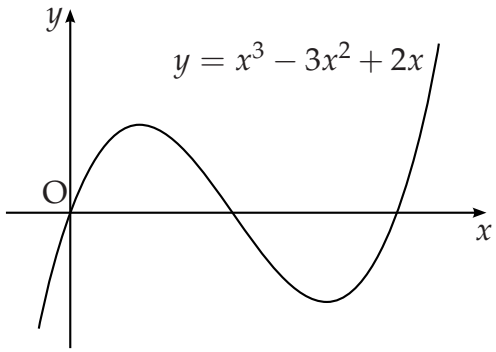
Prove that this curve has no stationary points. **5**

[SQA] 12. Find the coordinates of the turning points of the curve with equation $y = x^3 - 3x^2 - 9x + 12$ and determine their nature. **8**

[SQA] 13. The diagram shows a sketch of the graph of $y = x^3 - 3x^2 + 2x$.

(*a*) Find the equation of the tangent to this curve at the point where $x = 1$. **5**

(*b*) The tangent at the point $(2, 0)$ has equation $y = 2x - 4$. Find the coordinates of the point where this tangent meets the curve again. **5**



[SQA] 14. Find the equation of the tangent to the curve $y = 2\sin(x - \frac{\pi}{6})$ at the point where $x = \frac{\pi}{3}$. **4**

[SQA] 15. Differentiate $\sin 2x + \dfrac{2}{\sqrt{x}}$ with respect to x. **4**

[SQA] 16. Find $\frac{dy}{dx}$ given that $y = \sqrt{1 + \cos x}$. **3**

17. Given that $f(x) = (4 - 3x^2)^{-\frac{1}{2}}$ on a suitable domain, find $f'(x)$.

A. $-3x(4 - 3x^2)^{-\frac{1}{2}}$

B. $-\frac{1}{2}(4 - 6x)^{-\frac{3}{2}}$

C. $2(4 - 3x^3)^{\frac{1}{2}}$

D. $3x(4 - 3x^2)^{-\frac{3}{2}}$ **2**

[SQA] 18. Given that $f(x) = (5x - 4)^{\frac{1}{2}}$, evaluate $f'(4)$. **3**

[SQA] 19. If $f(x) = \cos^2 x - \frac{2}{3x^2}$, find $f'(x)$. **4**

[SQA] 20. Differentiate $4\sqrt{x} + 3\cos 2x$ with respect to x. **4**

[END OF QUESTIONS]