Easter Differentiation Solutions

1) $f(x) = \dfrac{x-1}{\sqrt{x}}$

$= \dfrac{x-1}{x^{1/2}}$

$= x^{1/2} - x^{-1/2}$

$f'(x) = \dfrac{1}{2}x^{-1/2} + \dfrac{1}{2}x^{-3/2}$

$= \dfrac{1}{2\sqrt{x}} + \dfrac{1}{2\sqrt{x^3}}$

$f'(4) = \dfrac{1}{2\sqrt{4}} + \dfrac{1}{2\sqrt{4^3}}$

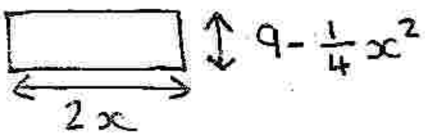
$= \dfrac{1}{4} + \dfrac{1}{16}$

$= \underline{\underline{\dfrac{5}{16}}}$

2) $y = 3x^{-2} + 2x^{3/2}$

$\underline{\underline{\dfrac{dy}{dx} = -6x^{-3} + 3x^{1/2}}}$

3a)



The question should ask you to prove the area is $18x - \frac{1}{2}x^3$, not x^2 !!

Area $= 2x\left(9 - \frac{1}{4}x^2\right)$

$= \underline{\underline{18x - \frac{1}{2}x^3}}$

3b) $A = 18x - \frac{1}{2}x^3$

$V = 60\left(18x - \frac{1}{2}x^3\right)$

$= 1080x - 30x^3$

$\frac{dV}{dx} = 1080 - 90x^2$

For stat pts, $\frac{dV}{dx} = 0$

$1080 - 90x^2 = 0$

$1080 = 90x^2$

$x^2 = 12$

$x = \pm\sqrt{12}$

$x = \pm 2\sqrt{3}$

However as x is a length, we know $x = 2\sqrt{3}$

x	$\rightarrow$	$2\sqrt{3}$	$\rightarrow$
$\frac{dV}{dx}$	$+$	0	$-$
slope	/	—	\

max tp at $x = 2\sqrt{3}$, giving

$V = 1080 \times 2\sqrt{3} - 30(2\sqrt{3})^3$

$= 2160\sqrt{3} - 720\sqrt{3}$

$= \underline{\underline{1440\sqrt{3}}}$ m^3

4) $f(x) = \frac{1}{\sqrt[5]{x}}$

$= x^{-1/5}$

$\underline{\underline{f'(x) = -\frac{1}{5}x^{-6/5}}}$

5) $y = 5x^3 - 12x$

$\frac{dy}{dx} = 15x^2 - 12$

At $(1, -7)$

$m = 15(1^2) - 12$

$\underline{\underline{m = 3}}$

6) $y = 5x^2 + 2$

$\frac{dy}{dx} = 10x$

At $(-1, 7)$

$m = 10 \times (-1)$

$m = -10$

so $y - 7 = -10(x - (-1))$

$y - 7 = -10x - 10$

$\underline{\underline{y = -10x - 3}}$

7a) $y = x^3 - 9x + 4$

$\frac{dy}{dx} = 3x^2 - 9$

if $m = 3$,

$3x^2 - 9 = 3$

$3x^2 = 12$

$x^2 = 4$

$x = \pm 2$

if $x = 2$, $y = 2^3 - 9(2) + 4$

$= 8 - 18 + 4$

$= -6 \rightarrow (2, -6)$

so $y + 6 = 3(x - 2)$

$y + 6 = 3x - 6$

$y = 3x - 12$

if $x = -2$, $y = (-2)^3 - 9(-2) + 4$

$= -8 + 18 + 4$

$= 14 \rightarrow (-2, 14)$

so $y - 14 = 3(x + 2)$

$y - 14 = 3x + 6$

$\underline{\underline{y = 3x + 20}}$

$\underline{\underline{y = 3x - 12}}$

7b) Shortest distance can be represented by a line perp. to both tangents Consider this line positioned to pass through (0,0). Let A be the point where it meets $y=3x-12$ and B be the place it meets $y=3x+20$. Since they have gradient 3, it has gradient $-\frac{1}{3}$ and eqn

$$y-0=-\frac{1}{3}(x-0)$$
$$y=-\frac{1}{3}x$$

At A

$$-\frac{1}{3}x=3x-12$$
$$-x=9x-36$$
$$-10x=-36$$
$$x=\frac{36}{10}$$
$$x=\frac{18}{5}$$
$$y=-\frac{1}{3}\times\frac{18}{5}$$
$$=-\frac{18}{15}$$
$$=-\frac{6}{5}\rightarrow A\left(\frac{18}{5},-\frac{6}{5}\right)$$

At B

$$-\frac{1}{3}x=3x+20$$
$$-x=9x+60$$
$$-10x=60$$
$$x=-6$$
$$y=-\frac{1}{3}\times(-6)$$
$$=2\rightarrow B(-6,2)$$

distance from A to B

$$=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$=\sqrt{\left(-6-\frac{18}{5}\right)^2+\left(2+\frac{6}{5}\right)^2}$$
$$=\sqrt{\left(-\frac{48}{5}\right)^2+\left(\frac{16}{5}\right)^2}$$
$$=\sqrt{\frac{2304}{25}+\frac{256}{25}}$$
$$=\sqrt{\frac{2560}{25}}$$
$$=\frac{16\sqrt{10}}{5}$$

8) $y = x - \frac{16}{\sqrt{x}}$

$= x - 16x^{-1/2}$

$\frac{dy}{dx} = 1 + 8x^{-3/2}$

$= 1 + \frac{8}{\sqrt{x^3}}$

when $x = 4$

$m = 1 + \frac{8}{\sqrt{4^3}}$

$m = 1 + \frac{8}{8}$

$m = 2$

if $x = 4$

$y = 4 - \frac{16}{\sqrt{4}}$

$y = 4 - 8$

$y = -4 \Rightarrow (4, -4)$

$y + 4 = 2(x - 4)$

$y + 4 = 2x - 8$

$\underline{y = 2x - 12}$

9)

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = 4\pi r^2$

when $r = 2$

$\frac{dV}{dr} = 4\pi \times 2^2$

$= \underline{16\pi}$

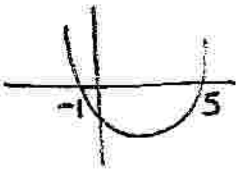
10) $f(x) = \frac{1}{3}x^3 - 2x^2 - 5x - 4$

$f'(x) = x^2 - 4x - 5$

$f'(x) = 0$ for roots $\Rightarrow$ $x^2 - 4x - 5 = 0$

$(x-5)(x+1) = 0$

$x = 5$ or $x = -1$



$f(x)$ is increasing when $f'(x) > 0$

ie $\underline{x < -1}$ and $\underline{x > 5}$

11) $y = 2x^3 + 3x^2 + 4x - 5$

$\frac{dy}{dx} = 6x^2 + 6x + 4$

For SPs $\frac{dy}{dx} = 0$

ie $6x^2 + 6x + 4 = 0$

$2(3x^2 + 3x + 2) = 0$

$b^2 - 4ac = 3^2 - 4 \times 3 \times 2$

$= 9 - 24$

$= -15$

Since $b^2 - 4ac < 0$ there are no real solutions to $\frac{dy}{dx} = 0$ and therefore the curve has no SPs

12) $y = x^3 - 3x^2 - 9x + 12$

$\frac{dy}{dx} = 3x^2 - 6x - 9$

For SPs $\frac{dy}{dx} = 0$

ie $3x^2 - 6x - 9 = 0$

$3(x^2 - 2x - 3) = 0$

$3(x-3)(x+1) = 0$

$x = 3$ or $x = -1$

if $x = 3$

$y = 3^3 - 3(3^2) - 9(3) + 12$

$= 27 - 27 - 27 + 12$

$= -15 \rightarrow (3, -15)$

if $x = -1$

$y = (-1)^3 - 3(-1)^2 - 9(-1) + 12$

$= -1 - 3 + 9 + 12$

$= 17 \rightarrow (-1, 17)$

x	$\rightarrow$	-1	$\rightarrow$	3	$\rightarrow$
$\frac{dy}{dx}$	$+$	0	$-$	0	$+$
slope	/	—	\	—	/

max tp at $(-1, 17)$

min tp at $(3, -15)$

13a) $y = x^3 - 3x^2 + 2x$

$\frac{dy}{dx} = 3x^2 - 6x + 2$

when $x = 1$

$m = 3(1^2) - 6(1) + 2$

$m = 3 - 6 + 2$

$m = -1$

and $y = 1^3 - 3(1^2) + 2(1)$

$= 1 - 3 + 2$

$= 0 \rightarrow (1, 0)$

$$y - 0 = -1(x - 1)$$

$$\underline{\underline{y = -x + 1}}$$

b) At pts of intersection

$x^3 - 3x^2 + 2x = 2x - 4$

$x^3 - 3x^2 + 4 = 0$

$$\begin{array}{r|rrrr} 2 & 1 & -3 & 0 & 4 \\ & & 2 & -2 & -4 \\ \hline & 1 & -1 & -2 & \underline{|0} \end{array}$$

$(x - 2)(x^2 - x - 2) = 0$

$(x - 2)(x - 2)(x + 1) = 0$

$x - 2 = 0$ or $x + 1 = 0$

$x = 2$ (twice) $\quad x = -1$

(point already given)

if $x = -1$

$y = 2(-1) - 4$

$= -2 - 4$

$= -6$

so required point is $\underline{\underline{(-1, -6)}}$

14) $y = 2\sin(x - \frac{\pi}{6})$

$\frac{dy}{dx} = 2\cos(x - \frac{\pi}{6}) \times 1$

at $x = \frac{\pi}{3}$

$m = 2\cos(\frac{\pi}{3} - \frac{\pi}{6})$

$= 2\cos(\frac{\pi}{6})$

$= \sqrt{3}$

if $x = \frac{\pi}{3}$

$y = 2\sin(\frac{\pi}{3} - \frac{\pi}{6})$

$= 2\sin(\frac{\pi}{6})$

$= 1 \rightarrow (\frac{\pi}{3}, 1)$

$y - 1 = \sqrt{3}(x - \frac{\pi}{3})$

$y - 1 = \sqrt{3}x - \frac{\sqrt{3}\pi}{3}$

15) $f(x) = \sin 2x + \frac{2}{\sqrt{x}}$

$= \sin 2x + 2x^{-1/2}$

$f'(x) = \cos 2x \times 2 - x^{-3/2}$

$= 2\cos 2x - \frac{1}{\sqrt{x^3}}$

16) $y = \sqrt{1+\cos x} = (1+\cos x)^{1/2}$

$\frac{dy}{dx} = \frac{1}{2}(1+\cos x)^{-1/2} \times (-\sin x)$

$= \frac{1}{2\sqrt{1+\cos x}} \times -\sin x$

$= -\frac{\sin x}{2\sqrt{1+\cos x}}$

17) $f(x) = (4-3x^2)^{-1/2}$

$f'(x) = -\frac{1}{2}(4-3x^2)^{-3/2} \times (-6x)$

$= 3x(4-3x^2)^{-3/2}$

18) $f(x) = (5x-4)^{1/2}$

$f'(x) = \frac{1}{2}(5x-4)^{-1/2} \times 5$

$= \frac{5}{2\sqrt{5x-4}}$

$f'(4) = \frac{5}{2\sqrt{5\times 4-4}}$

$= \frac{5}{2\sqrt{16}}$

$= \frac{5}{8}$

19) $f(x) = \cos^2 x - \dfrac{2}{3x^2}$

$= \cos^2 x - \dfrac{2}{3}x^{-2}$

$f'(x) = 2\cos x \times -\sin x + \dfrac{4}{3}x^{-3}$

$= -2\sin x\cos x + \dfrac{4}{3x^3}$

$= -\sin 2x + \dfrac{4}{3x^3}$

20) $f(x) = 4\sqrt{x} + 3\cos 2x$

$= 4x^{1/2} + 3\cos 2x$

$f'(x) = 2x^{-1/2} - 3\sin 2x \times 2$

$= \dfrac{2}{\sqrt{x}} - 6\sin 2x$