EASTER GEOMETRY

(a) $x^2 + y^2 - 8x - 10y + 9 = 0$

Centre $(4,5)$

$[(x-4)^2 - 16] + [(y-5)^2 - 25] + 9 = 0$

$(x-4)^2 + (y-5)^2 + 9 - 41 = 0$

$(x-4)^2 + (y-5)^2 = 32$

Radius $= \sqrt{32}$

$= \sqrt{16}\sqrt{2}$

$= \underline{4\sqrt{2}}$ as required.

(b) Centre $(-2,-1)$ Point $(-4,1)$

$m_{radius} = \frac{1-(-1)}{-4-(-2)}$

$= \frac{2}{-2}$

$= -1$

$m_{tang} = 1$ Pt $(-4,1)$

$y - 1 = 1(x+4)$

$y - 1 = x + 4$

$\underline{y = x + 5}$

(c) $x^2 + y^2 - 8x - 10y + 9 = 0$

Sub $y = x + 5$ into this equation

$\Rightarrow x^2 + (x+5)^2 - 8x - 10(x+5) + 9 = 0$

$x^2 + x^2 + 10x + 25 - 8x - 10x - 50 + 9 = 0$

$2x^2 - 8x - 16 = 0$

$x^2 - 4x - 8 = 0$

$a = 1$, $b = -4$, $c = -8$

$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-8)}}{2}$

$x = \frac{4 \pm \sqrt{48}}{2}$

$x = \frac{4 \pm \sqrt{16}\sqrt{3}}{2}$

$x = \frac{4 \pm 4\sqrt{3}}{2}$

$\underline{x = 2 \pm 2\sqrt{3}}$

(ii) Distance between centres $(-2,-1)$ and $(4,5)$

$= \sqrt{(4-(-2))^2 + 5 - (-1)^2}$

$= \sqrt{72}$

$= \sqrt{36}\sqrt{2}$

$= \underline{6\sqrt{2}}$

Radius of circle P $= 4\sqrt{2}$

Radius of circle Q $= 2\sqrt{2}$

$r_P + r_Q = 4\sqrt{2} + 2\sqrt{2}$

$= 6\sqrt{2}$

= Distance between centres

$\therefore$ P and Q touch

② (a) (i) $x^2+y^2+14x+4y-19=0$ $\quad y=3-x$

$\Rightarrow x^2+(3-x)^2+14x+4(3-x)-19=0$

$x^2+9-6x+x^2+14x+12-4x-19=0$

$2x^2+4x+2=0$

$x^2+2x+1=0$

$(x+1)^2=0$

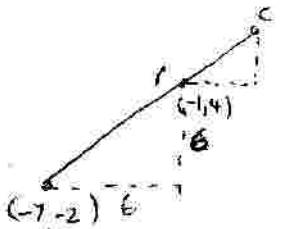
Equal roots so $y=3-x$ is a tangent to $x^2+y^2+14x+4y-19=0$

$\therefore$ one point of contact

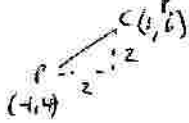
(ii) when $x=-1$, $y=3-(-1)$ point of contact $P(-1,4)$

$=4$

② (b)



If the radius is one third of the radius of the large circle, then by stepping out.



$\therefore C(1,6)$

radius $=\sqrt{2^2+2^2}$

$=\sqrt{8}$

$=2\sqrt{2}$

So $(x-1)^2+(y-6)^2=8$

③ C_1: $(x+1)^2+(y-1)^2=121$

Centre $(-1,1)$ radius $=\sqrt{121}$

$=11$

C_2: $x^2+y^2-4x+6y+p=0$

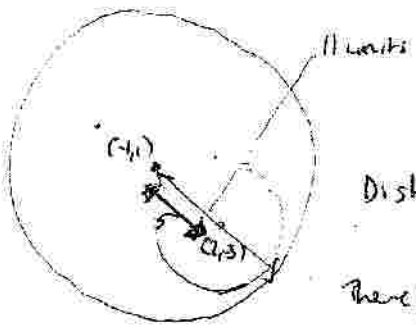
Centre $(2,-3)$

$(x-2)^2+(y+3)^2-4-9+p=0$

$(x-2)^2+(y+3)^2=13-p$

radius $=\sqrt{13-p}$

Since radius >0 by definition, $p<13$



Distance between two centres $=\sqrt{(-3-1)^2+(2-(-3))^2}$

$=5$

Therefore $\sqrt{13-p}<6$ $\quad (11-5)$

$13-p<36$

$13<36+p$

$-23<p$

$\therefore -23<p<13$