INTEGRATION

① $\int \frac{1}{3x^4}\,dx$

$= \int \frac{1}{3}x^{-4}\,dx$

$= \frac{\frac{1}{3}x^{-3}}{-3} + c$

$= \underline{\underline{-\frac{1}{9x^3} + c}}$ $\boxed{A}$

② $\int \frac{x^2-5}{x\sqrt{x}}\,dx$

$= \int \frac{x^2-5}{x^1 x^{\frac{1}{2}}}\,dx$

$= \int \frac{x^2-5}{x^{\frac{3}{2}}}\,dx$

$= \int \frac{x^2}{x^{\frac{3}{2}}} - \frac{5}{x^{\frac{3}{2}}}\,dx$

$= \int \frac{x^{\frac{4}{2}}}{x^{\frac{3}{2}}} - 5x^{-\frac{3}{2}}\,dx$

$= \int x^{\frac{1}{2}} - 5x^{-\frac{3}{2}}\,dx$

$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{5x^{-\frac{1}{2}}}{-\frac{1}{2}} + c$

$= \frac{2x^{\frac{3}{2}}}{3} + 10x^{-\frac{1}{2}} + c$

$= \underline{\underline{\frac{2\sqrt{x^3}}{3} + \frac{10}{\sqrt{x}} + c}}$

③ $\int_{-2}^{2} \overset{\text{TOP}}{(14-x^2)} - \overset{\text{BOTTOM}}{(2x^2+2)}\,dx$

$= \int_{-2}^{2} 12-3x^2\,dx$ $\boxed{C}$

④ $\int 6x^2 - x + \cos x\,dx$

$= \frac{6x^3}{3} - \frac{x^2}{2} + \sin x + c$

$= \underline{\underline{2x^3 + \sin x - \frac{x^2}{2} + c}}$

⑤ (a) $P(1,0)$ $Q(2,0)$

(b) $T.S.A = \int_0^1 x^3-x^2-4x+4\,dx + \left|\int_1^2 x^3-x^2-4x+4\,dx\right|$

$(x+2)(x-1)(x-2)$

$= (x+2)(x^2-3x+2)$

$= x^3-3x^2+2x+2x^2-6x+4$

$= x^3-x^2-4x+4$

$= \left[\frac{x^4}{4}-\frac{x^3}{3}-2x^2+4x\right]_0^1 + \left|\left[\frac{x^4}{4}-\frac{x^3}{3}-2x^2+4x\right]_1^2\right|$

$= \left[\frac{1}{4}-\frac{1}{3}-2+4\right]-[0] + \left|\left[\frac{16}{4}-\frac{8}{3}-8+8\right]-\left[\frac{1}{4}-\frac{1}{3}-2+4\right]\right|$

$\frac{23}{12} + \left|\left[\frac{4}{3}\right]-\left[\frac{23}{12}\right]\right|$

$\frac{23}{12} + \left|\frac{16}{12}-\frac{23}{12}\right|$

$\frac{23}{12} + \frac{7}{12}$

$\underline{\underline{\frac{30}{12}\text{ sq units}}}$ $\underline{\underline{(2\frac{1}{2}\text{ sq units})}}$

⑥ T.S.A
$$= \int_{-2}^{0} (x^3 - x^2 - 4x + 4) - (2x+4)\, dx$$
$$+ \int_{0}^{3} (2x+4) - (x^3 - x^2 - 4x + 4)\, dx$$

$$\int_{-2}^{0} x^3 - x^2 - 4x + 4 - (2x+4)\, dx$$
$$= \int_{-2}^{0} x^3 - x^2 - 6x\, dx$$
$$= \left[\frac{x^4}{4} - \frac{x^3}{3} - 3x^2\right]_{-2}^{0}$$
$$= [0] - \left[\frac{16}{4} + \frac{8}{3} - 12\right]$$
$$= [0] - \left[-\frac{24}{3} + \frac{8}{3}\right]$$
$$= \underline{\underline{\frac{16}{3} \text{ sq units}}}$$

$$\int_{0}^{3} (2x+4) - (x^3 - x^2 - 4x + 4)\, dx$$
$$= \int_{0}^{3} 6x - x^3 + x^2\, dx$$
$$= \left[3x^2 - \frac{x^4}{4} + \frac{x^3}{3}\right]_{0}^{3}$$
$$= \left[27 - \frac{81}{4} + \frac{27}{3}\right] - [0]$$
$$= \left[36 - \frac{81}{4}\right] - [0]$$
$$= \frac{144 - 81}{4}$$
$$= \underline{\underline{\frac{63}{4} \text{ sq units}}}$$

$$\text{T.S.A} = \frac{16}{3} + \frac{63}{4}$$
$$= \frac{64}{12} + \frac{189}{12}$$
$$= \underline{\underline{\frac{253}{12} \text{ sq units}}}$$

⑦ (a) $y = -(x-2)^2 + 4$
$$= \underline{\underline{4 - (x-2)^2}}$$

(b) Shaded area
$$= \int_{2}^{k} 4 - (x-2)^2\, dx$$
$$= \int_{2}^{k} 4 - [x^2 - 4x + 4]\, dx$$
$$= \int_{2}^{k} 4x - x^2\, dx$$
$$= \left[2x^2 - \frac{x^3}{3}\right]_{2}^{k}$$
$$= 2k^2 - \frac{k^3}{3} - \left[8 - \frac{8}{3}\right]$$

$$= -\frac{1}{3}k^3 + 2k^2 - \frac{16}{3}$$
as required

⑧ (a) $y = (2x-9)^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}(2x-9)^{-\frac{1}{2}} \cdot 2$

$= \frac{1}{\sqrt{2x-9}}$

when $x=9$, $\frac{dy}{dx} = \frac{1}{\sqrt{9}}$

$= \underline{\underline{\frac{1}{3}}}$

$\Rightarrow m_{TANGENT} = \frac{1}{3}$

when $x = 9$, $y = \sqrt{2(9)-9}$

$= \sqrt{9}$

$= \underline{\underline{3}}$

$m = \frac{1}{3}$, $pt(9,3)$

$y - 3 = \frac{1}{3}(x-9)$

$\underline{\underline{y = \frac{1}{3}x \text{ as required}}}$

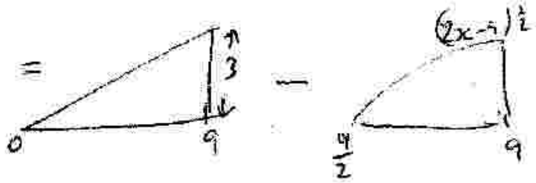
(b) A is the point at which $y=(2x-9)^{\frac{1}{2}}$ intersects with the x-axis, so at this point $y=0$

$\sqrt{2x-9} = 0$

$2x-9 = 0$

$\underline{\underline{x = \frac{9}{2}}}$ $\quad \underline{\underline{\therefore A(\frac{9}{2}, 0)}}$

(c) shaded area =



area $= \frac{1}{2} \times 9 \times 3$

$= \underline{\underline{\frac{27}{2} \text{ sq units}}}$

area $= \int_{\frac{9}{2}}^{9} (2x-9)^{\frac{1}{2}} dx$

$= \left[\frac{(2x-9)^{\frac{3}{2}}}{\frac{3}{2} \cdot 2}\right]_{\frac{9}{2}}^{9}$

$= \left[\frac{\sqrt{(2x-9)^3}}{3}\right]_{\frac{9}{2}}^{9}$

$= \left[\frac{\sqrt{9^3}}{3}\right] - \left[\frac{\sqrt{0^3}}{3}\right]$

$= \underline{\underline{9 \text{ sq units}}}$

$\therefore$ Total shaded area $= \frac{27}{2} - 9$

$= \frac{27}{2} - \frac{18}{2}$

$= \underline{\underline{\frac{9}{2} \text{ sq units}}}$

9 (a) $m = 3$
$n = 2$

(b) $-4\cos(2x)+3 = 3\cos 2x$

$7\cos 2x = 3$

$\boxed{\cos 2x = \frac{3}{7}}$ → Cos is pos

S	A 64.6°
T	C 360 − 64.6°

Acute angle
$= \cos^{-1}\left(\frac{3}{7}\right)$
$= 64.6°$

$2x = 64.6°, 295.4°$
$x = 32.3°, 147.7°$

Coordinates of points of intersection

when $x = 32.3°$, $y = 3\cos(64.6°)$
$= 3\left(\frac{3}{7}\right)$
$= \frac{9}{7}$

$\underline{\underline{(32.3°, 1.3)}}$

when $x = 147.7°$, $y = 3\cos(295.4°)$
$= 3\left(\frac{3}{7}\right)$
$= \frac{9}{7}$

$\underline{\underline{(147.7°, 1.3)}}$

N.B
In radians,
Points of intersection are
$(2.6, 1.3)$ and $(0.6, 1.3)$
To convert to radians,
$\times \pi \div 180$

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c) Shaded area $= \int_{0.6}^{2.6} -4\cos(2x)+3-(3\cos 2x)\,dx$

$= \int_{0.6}^{2.6} 3 - 7\cos 2x \, dx$

$= \left[3x - \frac{7}{2}\sin 2x\right]_{0.6}^{2.6}$

$= \left[7.8 - \frac{7}{2}\sin(5.2)\right] - \left[1.8 - \frac{7}{2}\sin(1.2)\right]$

$= 10.89 - [-1.46]$

$= \underline{\underline{12.35 \text{ sq units}}}$

(10) (a) P is the point of intersection of $y = x(x+3)$ and $y = \frac{4}{x^2}$

Q is the point of intersection of $y = x - \frac{1}{4}x^2$ and $y = \frac{4}{x^2}$

P: when $y = 4$, $4 = \frac{4}{x^2}$

$4x^2 = 4$

$x^2 = 1$

$x = \pm 1$

$\therefore \underline{\underline{p = 1}}$

Q: when $y = 1$, $1 = \frac{4}{x^2}$

$x^2 = 4$

$x = \pm 2$

$\therefore \underline{\underline{q = 2}}$

(b) shaded area $= \int_0^1 x(x+3)\,dx + \int_1^2 \frac{4}{x^2}\,dx - \int_0^2 x - \frac{1}{4}x^2\,dx$

$\int_0^1 x^2 + 3x\,dx$

$= \left[\frac{x^3}{3} + \frac{3x^2}{2}\right]_0^1$

$= \left[\frac{1}{3} + \frac{3}{2}\right] - [0]$

$= \underline{\underline{\frac{11}{6}}}$

$\int_1^2 4x^{-2}\,dx$

$= \left[\frac{4x^{-1}}{-1}\right]_1^2$

$= \left[-\frac{4}{x}\right]_1^2$

$= [-2] - [-4]$

$= \underline{\underline{2}}$

$\int_0^2 x - \frac{1}{4}x^2\,dx$

$= \left[\frac{x^2}{2} - \frac{x^3}{12}\right]_0^2$

$= \left[2 - \frac{2}{3}\right] - [0]$

$= \underline{\underline{\frac{4}{3}}}$

$\therefore$ shaded area $= \frac{11}{6} + \frac{12}{6} - \frac{8}{6}$

$= \frac{15}{6}$

$= \underline{\underline{2\frac{1}{2} \text{ sq units}}}$