Easter Trigonometry

[SQA] 1. Solve $2\sin 3x^\circ - 1 = 0$ for $0 \leq x \leq 180$. **4**

2. Solve $2\cos x = \sqrt{3}$ for x, where $0 \leq x < 2\pi$.

   A. $\frac{\pi}{3}$ and $\frac{5\pi}{3}$

   B. $\frac{\pi}{3}$ and $\frac{2\pi}{3}$

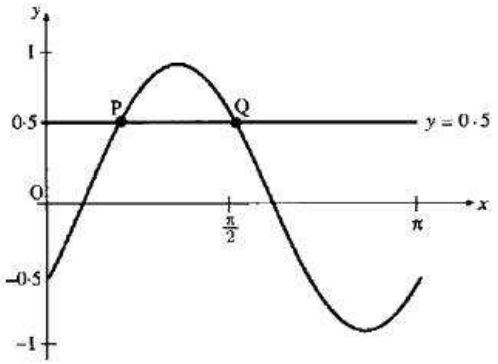
   C. $\frac{\pi}{6}$ and $\frac{5\pi}{6}$

   D. $\frac{\pi}{6}$ and $\frac{11\pi}{6}$ **2**

[SQA] 3. Find the exact solutions of the equation $4\sin^2 x = 1$, $0 \leq x < 2\pi$. **4**

[SQA] 4. The diagram shows a sketch of the graph of $y = \sin\left(2x - \frac{\pi}{6}\right)$, $0 \leq x \leq \pi$, and the straight line $y = 0{\cdot}5$. These graphs intersect at P and Q.

Find algebraically the coordinates of P and Q. **4**



5. If the exact value of $\cos x$ is $\frac{1}{\sqrt{5}}$, find the exact value of $\cos 2x$.

   A. $-\frac{3}{5}$

   B. $-\frac{2}{\sqrt{5}}$

   C. $\frac{2}{\sqrt{5}}$

   D. $\frac{3}{5}$ **2**

[SQA] 6. Given that $\cos D = \frac{2}{\sqrt{5}}$ and $0 < D < \frac{\pi}{2}$, find the exact values of $\sin D$ and $\cos 2D$. **3**

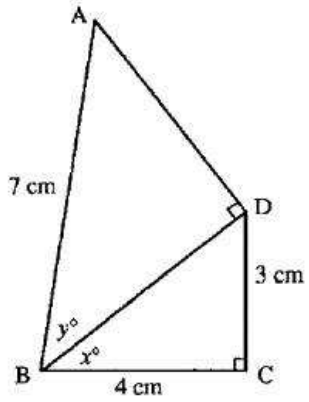
[SQA] 7. If $\cos\theta = \frac{4}{5}$, $0 \leq \theta < \frac{\pi}{2}$, find the exact value of

(*a*) $\sin 2\theta$ **2**

(*b*) $\sin 4\theta$. **3**

[SQA] 8. **The diagram shows two right-angled triangles ABD and BCD with AB = 7cm, BC = 4cm and CD = 3cm. Angle DBC = $x°$ and angle ABD = $y°$.**

Show that the exact value of $\cos(x+y)°$ is $\frac{20-6\sqrt{6}}{35}$. **3**



[SQA] 9.

(*a*) Show that $2\cos 2x° - \cos^2 x° = 1 - 3\sin^2 x°$. **2**

(*b*) Hence solve the equation $2\cos 2x° - \cos^2 x° = 2\sin x°$ in the interval $0 \leq x < 360$. **4**

[SQA] 10. Solve the equation $3\cos 2x° + \cos x° = -1$ in the interval $0 \leq x \leq 360$. **5**

[SQA] 11. Functions f and g are defined on suitable domains by $f(x) = \sin(x°)$ and $g(x) = 2x$.

(*a*) Find expressions for:

(i) $f(g(x))$;

(ii) $g(f(x))$. **2**

(*b*) Solve $2f(g(x)) = g(f(x))$ for $0 \leq x \leq 360$. **5**

[SQA] 12. Solve the equation $\cos 2x° + 5\cos x° - 2 = 0$, $0 \leq x < 360$. **5**

[SQA] 13. Find the values of t, where $0 < t < 2\pi$, for which $4\cos\left(2t - \frac{\pi}{4}\right)$ has its maximum value. **4**

[SQA] 14. (a) Express $\sin x° - 3\cos x°$ in the form $k\sin(x-a)°$ where $k > 0$ and $0 \le a < 360$. Find the values of k and a. **4**

(b) Find the maximum value of $5 + \sin x° - 3\cos x°$ and state a value of x for which this maximum occurs. **2**

[SQA] 15. The displacement, d units, of a wave after t seconds, is given by the formula $d = \cos 20t° + \sqrt{3}\sin 20t°$.

(a) Express d in the form $k\cos(20t° - \alpha°)$, where $k > 0$ and $0 \le \alpha \le 360$. **4**

(b) Sketch the graph of d for $0 \le t \le 18$. **4**

(c) Find, correct to one decimal place, the values of t, $0 \le t \le 18$, for which the displacement is 1·5 units. **3**

[SQA] 16. Find the maximum value of $\cos x - \sin x$ and the value of x for which it occurs in the interval $0 \le x \le 2\pi$. **6**

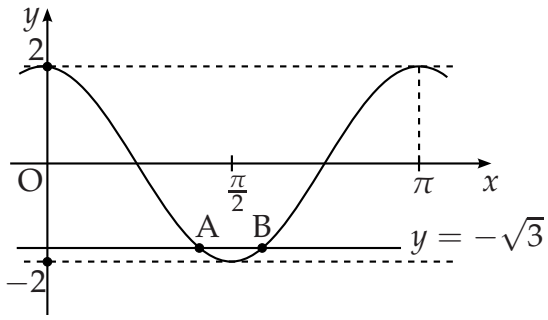
[SQA] 17. Solve the equation $2\sin x° - 3\cos x° = 2{\cdot}5$ in the interval $0 \le x < 360$. **8**

[SQA] 18. The diagram shows the graph of a cosine function from 0 to π.

(a) State the equation of the graph. **1**

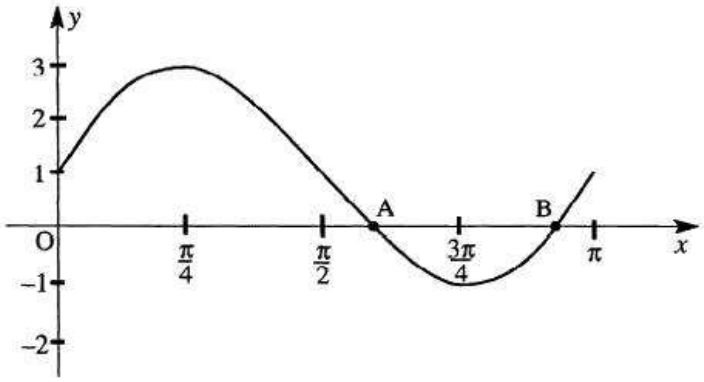
(b) The line with equation $y = -\sqrt{3}$ intersects this graph at point A and B.

Find the coordinates of B. **3**



[SQA] 19. Find, correct to one decimal place, the value of x between 180 and 270 which satisfies the equation $3\cos(2x° - 40°) - 1 = 0$. **5**

[SQA] 20. The diagram below shows the graph of $y = 2\sin 2x + 1$ for $0 \le x \le \pi$.



(*a*) Find the coordinates of A and B (as shown in the diagram) by solving an appropriate equation algebraically. **(5)**

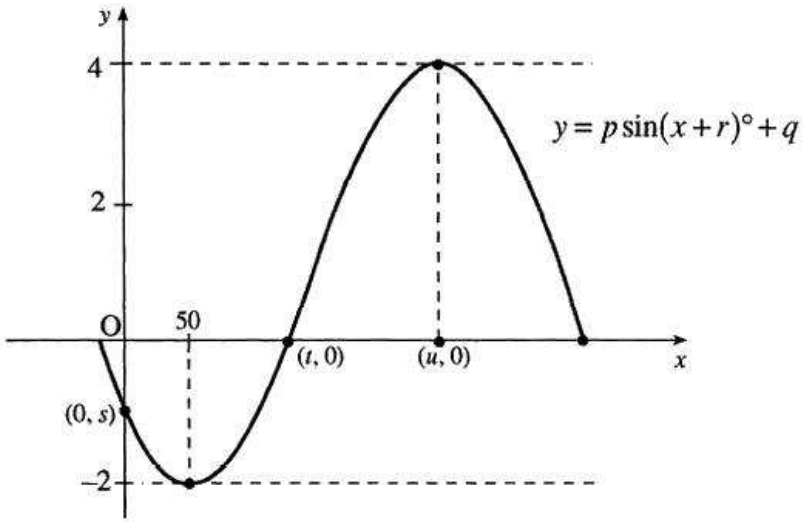
(*b*) The points (0, 2) and (π, 0) are joined by a straight line *l*. In how many points does *l* intersect the given graph ? **(1)**

(*c*) C is the point on the given graph with an *x*-coordinate of $\frac{\pi}{2}$. Explain whether C is above, below or on the line *l*. **(3)**

[SQA] 21. The sketch represents part of the graph of a trigonometric function of the form $y = p\sin(x + r)° + q$. It crosses the axes at (0, *s*) and (*t*,0), and has turning points at (50, –2) and (*u*, 4).

(i) Write down values for *p*, *q*, *r* and *u*. **(4)**

(ii) Find the values for *s* and *t*. **(4)**



[END OF QUESTIONS]