EASTER TRIGONOMETRY

(1) $2\sin 3x^\circ - 1 = 0$

$\boxed{\sin 3x^\circ = \frac{1}{2}}$ $\xrightarrow{\text{sin is pos}}$ S A / T C   (180 − 30, 30°)

Acute angle $= \sin^{-1}\left(\frac{1}{2}\right)$

$= 30^\circ$ +360°   +3

$3x^\circ = 30^\circ, 150^\circ, 390^\circ, 510^\circ$

$x^\circ = 10^\circ, 50^\circ, 130^\circ, 170^\circ$

(2) $2\cos x = \sqrt{3}$

$\boxed{\cos x = \frac{\sqrt{3}}{2}}$ $\xrightarrow{\text{cos is pos}}$ S A / T C   (30°, 360 − 30°)

Acute angle $= 30^\circ$

$x = 30^\circ, 330^\circ$

In radians, $x = \frac{30\pi}{180}, \frac{330\pi}{180}$

$\boxed{D}$

$x = \frac{\pi}{6}, \frac{11\pi}{6}$

(3) $4\sin^2 x = 1$

$\sin^2 x = \frac{1}{4}$

$\sin x = \pm\sqrt{\frac{1}{4}}$

$\boxed{\sin x = \pm\frac{1}{2}}$ $\xrightarrow{\text{sin is pos and sin is neg}}$ S A / T C   ($\pi -$, $\pi +$, $2\pi -$)

Acute angle $= \frac{\pi}{6}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

(4) $\boxed{\sin\left(2x - \frac{\pi}{6}\right) = 0.5}$ $\xrightarrow{\text{sin is pos}}$ S A / T C   ($\pi -$, $\frac{\pi}{6}$)

Acute angle $= \sin^{-1}\left(\frac{1}{2}\right)$

$= 30^\circ$

In radians, acute angle $= \frac{\pi}{6}$

So, $2x - \frac{\pi}{6} = \frac{\pi}{6}, \frac{5\pi}{6}$

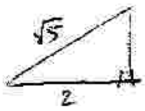
$2x = \frac{\pi}{3}, \pi$

$x = \frac{\pi}{6}, \frac{\pi}{2}$

When $x = \frac{\pi}{6}, y = 0.5$ and when $x = \frac{\pi}{2}, y = 0.5$

$\therefore P\left(\frac{\pi}{6}, 0.5\right) \quad Q\left(\frac{\pi}{2}, 0.5\right)$

(5)



$\cos x = \frac{1}{\sqrt{5}}$

$\cos 2x = 2\cos^2 x - 1$

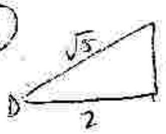
$= 2\left(\frac{1}{\sqrt{5}}\right)^2 - 1$

$= 2\left(\frac{1}{5}\right) - 1$

$= -\frac{3}{5}$

$\boxed{A}$

(6)



$\sin D = \frac{1}{\sqrt{5}}$

$\cos 2D = 2\cos^2 D - 1$

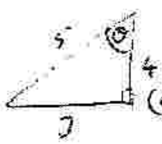
$= 2\left(\frac{2}{\sqrt{5}}\right)^2 - 1$

$= 2\left(\frac{4}{5}\right) - 1$

$= \frac{8}{5} - 1$

$= \frac{3}{5}$

(7)



$\sin\theta = \frac{3}{5} \quad \cos\theta = \frac{4}{5}$

(a) $\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right)$

$= \frac{24}{25}$

$\cos 2\theta = 2\cos^2\theta - 1$

$= 2\left(\frac{4}{5}\right)^2 - 1$

$= \frac{32}{25} - 1$

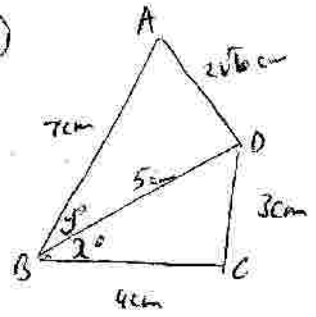
$= \frac{7}{25}$

(b) $\sin 4\theta = 2\sin(2\theta)\cos(2\theta)$

$= 2\left(\frac{24}{25}\right)\left(\frac{7}{25}\right)$

$= \frac{336}{625}$

(8)



BD = 5 units (3,4,5 Δ)

$AD^2 = 7^2 - 5^2$

$AD^2 = 24$

$AD = \sqrt{24}$

$AD = \sqrt{4}\sqrt{6}$

$= 2\sqrt{6}$

$\cos(x+y) = \cos x \cos y - \sin x \sin y$

$\cos x° = \frac{4}{5}$ $\sin x° = \frac{3}{5}$

$\cos y° = \frac{5}{7}$ $\sin y° = \frac{2\sqrt{6}}{7}$

So $\cos(x+y) = \left(\frac{4}{5}\right)\left(\frac{5}{7}\right) - \left(\frac{3}{5}\right)\left(\frac{2\sqrt{6}}{7}\right)$

$= \frac{20}{35} - \frac{6\sqrt{6}}{35}$

$= \frac{20 - 6\sqrt{6}}{35}$ as required

Identities:
- $\sin^2 x + \cos^2 x = 1$
- $\cos 2x = 1 - 2\sin^2 x$

(9) (a) $2\cos 2x° - \cos^2 x$

$= 2(1 - 2\sin^2 x) - (1 - \sin^2 x)$

$= 2 - 4\sin^2 x - 1 + \sin^2 x$

$= 1 - 3\sin^2 x$ as required

a b) $2\cos 2x° - \cos^2 x = 2\sin x$

$\Rightarrow 1 - 3\sin^2 x = 2\sin x$

$3\sin^2 x + 2\sin x - 1 = 0$

$(3\sin x - 1)(\sin x + 1) = 0$

180 − 19.5   19.5

S	A
T	C

sin is +ve

$\boxed{\sin x = \frac{1}{3}}$

Acute angle $= \sin^{-1}\left(\frac{1}{3}\right)$

$= 19.5°$

$x = 19.5°, 160.5°$

$\sin x = -1$

$x = 270°$

$\therefore x = 19.5°, 160.5°, 270°$

(10) $3\cos 2x° + \cos x° = -1$

$3\cos 2x° + \cos x° + 1 = 0$

$3(2\cos^2 x - 1) + \cos x° + 1 = 0$

$6\cos^2 x - 3 + \cos x° + 1 = 0$

$6\cos^2 x + \cos x - 2 = 0$

$(3\cos x + 2)(2\cos x - 1) = 0$

$\boxed{\cos x = -\frac{2}{3}}$

Acute angle $= \cos^{-1}\left(\frac{2}{3}\right)$

$= 48°$

180 − 48°   48°

S	A
T	C

180 + 48°

$x = 132°, 228°$

$\boxed{\cos x = \frac{1}{2}}$

Acute angle = 60°

60°

S	A
T	C

360 − 60°

$x = 60°, 300°$

$\therefore x = 60, 132°, 228°, 300°$

(11) (a) (i) $f(g(x))$

$= f(2x)$

$= \sin(2x°)$

(ii) $g(f(x))$

$= g(\sin(x°))$

$= 2\sin x°$

(b) $2\sin 2x° = 2\sin x°$

$\sin 2x° = \sin x°$

$\sin 2x° - \sin x° = 0$

$2\sin x \cos x - \sin x° = 0$

$\sin x°(2\cos x° - 1) = 0$

$\sin x = 0$

$x = 0, 360°$
180°

$\boxed{\cos x° = \frac{1}{2}}$

Acute angle = 60°

$x° = 60°, 300°$

$\therefore x = 0, 60°, 180°, 300°, 360°$

(12)

$$\cos 2x^\circ + 5\cos x^\circ - 2 = 0$$
$$2\cos^2 x - 1 + 5\cos x^\circ - 2 = 0$$
$$2\cos^2 x + 5\cos x - 3 = 0$$
$$(2\cos x - 1)(\cos x + 3) = 0$$

$\boxed{\cos x = \frac{1}{2}}$ — Acute angle $= 60^\circ$, $\underline{\underline{x^\circ = 60^\circ, 300^\circ}}$

$\cos x = -3$ — No real solutions as $-1 \leq \cos x \leq 1$

(13) The maximum value of $4\cos(2t - \frac{\pi}{4})$ is 4, when

$$\boxed{\cos(2t - \tfrac{\pi}{4}) = 1}$$
$$2t - \frac{\pi}{4} = 0^\circ, 2\pi$$
$$2t = \frac{\pi}{4}, \frac{9\pi}{4}$$
$$\underline{\underline{t = \frac{\pi}{8}, \frac{9\pi}{8}}}$$

(14) (a) $\sin x^\circ - 3\cos x^\circ$

$k\sin x^\circ\cos a^\circ - k\cos x^\circ\sin a$

$k\sin(x - a)^\circ = k\sin x\cos a - k\cos x\sin a$

$-k\sin a = -3$ $\quad \boxed{\begin{matrix} k\sin a = 3 \\ k\cos a = 1 \end{matrix}}$

$k^2 = 3^2 + 1^2$, $\underline{\underline{k = \sqrt{10}}}$

$\underline{\underline{\sin x^\circ - 3\cos x^\circ = \sqrt{10}\sin(x - 72)^\circ}}$

$a = \tan^{-1}\left(\frac{3}{1}\right)$

$\underline{\underline{a = 72^\circ}}$

(b). The maximum value of $5 + \sin x^\circ - 3\cos x^\circ$ is $5 + \sqrt{10}$ and it occurs when

$$\boxed{\sin(x - 72)^\circ = 1}$$
$$x - 72 = 90^\circ$$
$$\underline{\underline{x = 162^\circ}}$$

(15)(a) $d = \cos 20t^\circ + \sqrt{3}\sin 20t^\circ$

$k\cos(20t^\circ - \alpha^\circ) = k\cos 20t^\circ\cos\alpha^\circ + k\sin 20t^\circ\sin\alpha^\circ$

$\cos 20t^\circ \quad + \sqrt{3}\sin 20t^\circ$

$k\cos\alpha = 1 \qquad k\sin\alpha = \sqrt{3}$

$k\sin\alpha = \sqrt{3}$

$k\cos\alpha = 1$

$k^2 = (\sqrt{3})^2 + 1^2$, $\underline{\underline{k = 2}}$

$\tan\alpha = \frac{\sqrt{3}}{1}$

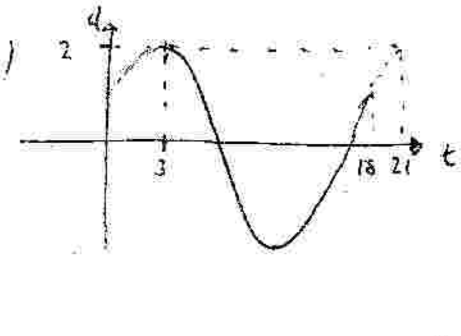
$\alpha = \tan^{-1}\sqrt{3}$

$\alpha = 60^\circ$

So $\underline{\underline{d = 2\cos(20t^\circ - 60^\circ)}}$

$2\cos(20t^\circ - 60^\circ)$
$= 2\cos 20(t^\circ - 3^\circ)$

(b)



(c) $2\cos(20t^\circ - 60)^\circ = 1.5$

$$\boxed{\cos(20t^\circ - 60^\circ) = \frac{1.5}{2}}$$ $\xrightarrow{\text{cosine +ve}}$ S | A 41.4, T | C 360 − 41.4

Acute angle $= \cos^{-1}(0.75)$
$= 41.4^\circ$

So $20t^\circ - 60^\circ = 41.4^\circ, 318.6^\circ$

$20t^\circ = 101.4^\circ, 378.6^\circ$

$\underline{\underline{t = 5.1 \text{ (one d.p.)}}}$

(16) $\cos x^\circ - \sin x^\circ$

$k\cos x^\circ \cos\alpha - k\sin x^\circ \sin\alpha$

$k\cos(x+\alpha) = k\cos x\cos\alpha - k\sin x\sin\alpha$

$k\sin\alpha = 1$ $\quad k^2 = 1^2 + 1^2 \quad \tan\alpha = \frac{1}{1}$

$k\cos\alpha = 1$ $\quad k^2 = 2 \quad \alpha = \tan^{-1}(1)$

$k = \sqrt{2} \quad \underline{\alpha = 45^\circ}$

In radians, $\alpha = \frac{\pi}{4}$

So $\cos x^\circ - \sin x^\circ = \sqrt{2}\cos\left(x + \frac{\pi}{4}\right)$ has a maximum value of $\sqrt{2}$ when $\boxed{\cos\left(x+\frac{\pi}{4}\right) = 1}$

$x + \frac{\pi}{4} = 0,\ 2\pi$

$x = \cancel{-\frac{\pi}{4}},\ \underline{\frac{7\pi}{4}}$

out with range

(17) $2\sin x^\circ - 3\cos x^\circ = 2.5$

$k\sin x^\circ \cos\alpha^\circ - k\cos x^\circ \sin\alpha^\circ$ $\quad (k\sin(x-\alpha))$

$\boxed{k\sin\alpha = 3 \quad k\cos\alpha = 2}$ $\quad k^2 = 3^2 + 2^2 \quad k^2 = 13 \quad k = \sqrt{13}$

So $2\sin x^\circ - 3\cos x^\circ = \sqrt{13}\sin(x - 56.3)^\circ$

$\alpha = \tan^{-1}\left(\frac{3}{2}\right) = \underline{56.3^\circ}$

$\Rightarrow \sqrt{13}\sin(x-56.3)^\circ = 2.5$

$\boxed{\sin(x-56.3)^\circ = \frac{2.5}{\sqrt{13}}} \xrightarrow{\text{sin is +ve}}$ $180 - 43.9 \quad 43.9$

S	A
T	C

Acute angle $= \sin^{-1}\left(\frac{2.5}{\sqrt{13}}\right) = 43.9^\circ$

$x - 56.3 = 43.9,\ 136.1$

$\underline{x = 100.2,\ 192.4^\circ}$

(18) (a) $y = 2\cos 2x$

(b) $2\cos 2x = -\sqrt{3}$

$\boxed{\cos 2x = -\frac{\sqrt{3}}{2}} \xrightarrow{\text{cos is -ve}}$ $\pi - \frac{\pi}{6}$, $\pi + \frac{\pi}{6}$

S	A
T	C

Acute angle $= \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

$2x = \frac{5\pi}{6},\ \frac{7\pi}{6}$

$x = \frac{5\pi}{12},\ \frac{7\pi}{12}$

$\therefore \underline{B\left(\frac{7\pi}{12}, -\sqrt{3}\right)}$

(19) $3\cos(2x^\circ - 40^\circ) - 1 = 0$

$\boxed{\cos(2x^\circ - 40^\circ) = \frac{1}{3}} \xrightarrow{\text{cos is +ve}}$ 70, 289

S	A
T	C

Acute angle $= \cos^{-1}\left(\frac{1}{3}\right) = 70.5^\circ$

$\Rightarrow 2x^\circ - 40^\circ = 70.5^\circ,\ 289.5^\circ,\ 430.5^\circ$

$2x^\circ = 110.5^\circ,\ 329.5^\circ,\ 470.5^\circ$

$x^\circ = 55.25^\circ,\ \cancel{164.8^\circ},\ 235.3$

out with range, out with range

20. (a) $2\sin 2x + 1 = 0$

$2\sin 2x = -1$

$\boxed{\sin 2x = -\frac{1}{2}}$ $\xrightarrow{\text{sin is neg}}$ S | A, T | C

Acute angle $= \sin^{-1}\left(\frac{1}{2}\right)$

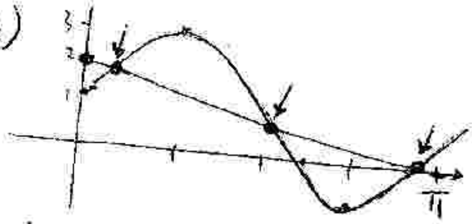
$= \frac{\pi}{6}$

$\pi + \frac{\pi}{6}$ $2\pi - \frac{\pi}{6}$

$2x = \frac{7\pi}{6}, \frac{11\pi}{6}$ $\therefore A\left(\frac{7\pi}{12}, 0\right)$

$x = \frac{7\pi}{12}, \frac{11\pi}{12}$ $B\left(\frac{11\pi}{12}, 0\right)$

(b)



l intersects the graph at $\underline{3}$ points

The equation of the graph is $y = 2\sin 2x + 1$

when $x = \frac{\pi}{2}$, $y = 2\sin(\pi) + 1$

$= 0 + 1$

$= \underline{1}$

(c) The equation of l is

$y = -\frac{2}{\pi}x + 2$

when $x = \frac{\pi}{2}$, $y = -\frac{2}{\pi}\left(\frac{\pi}{2}\right) + 2$

$y = -1 + 2$

$\underline{y = 1}$

$\therefore$ C is on the line l, since when $x = \frac{\pi}{2}$ the y-values of both line and curve are the same.

21. $y = p\sin(x + r)^\circ + q$

(i) • reflected in x-axis, amplitude of 3 $\Rightarrow p = -3$

• vertical shift of one unit up $\Rightarrow q = 1$

• horizontal phase shift of 40° left $\Rightarrow r = 40$

(ii) $\underline{t}$

$-3\sin(x+40)^\circ + 1 = 0$

$-3\sin(x+40)^\circ = -1$

$\boxed{\sin(x+40)^\circ = \frac{1}{3}}$ $\xrightarrow{\text{sin is pos}}$ S | A, T | C

$180 - 19.5$ 19.5

Acute angle $= \sin^{-1}\left(\frac{1}{3}\right)$

$= 19.5^\circ$

$x + 40 = 19.5, 160.5^\circ$

$x = -20.5, 120.5^\circ$

$\therefore$ ~~t is the point~~ $\underline{(120.5^\circ, 0)}$

$\underline{t = 120.5^\circ}$

$\underline{s}$ $(x = 0)$

$y = -3\sin(0+40)^\circ + 1$

$y = -3\sin(40)^\circ + 1$

$y = -0.9$

So $\underline{s = -0.9}$