N5

Elgin Academy

Prelim Examination 2013 / 14

MATHEMATICS

National Qualifications - National 5

Paper 1 (non-calculator)

Covering all three Units

Time allowed - 1 hour

Read carefully what is printed below

Total marks - 40

1. You may NOT use a calculator.
2. Use **blue** or **black** ink. Pencil may be used for graphs and diagrams only.
3. Write your working and answers on the blank paper provided. Write clearly the number of the question you are attempting. Extra paper may be requested at any time from the invigilator.
4. Square ruled paper is also provided.
5. Full credit will be given only where the solution contains appropriate working.
6. State the units for your answer where appropriate.

FORMULAE LIST

The roots of $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$ or $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

Area of a triangle: Area = $\frac{1}{2} ab \sin C$

Volume of a sphere: Volume = $\dfrac{4}{3}\pi r^3$

Volume of a cone: Volume = $\dfrac{1}{3}\pi r^2 h$

Volume of a Pyramid: Volume = $\dfrac{1}{3}Ah$

Standard deviation: $s = \sqrt{\dfrac{\sum(x - \bar{x})^2}{n-1}} = \sqrt{\dfrac{\sum x^2 - (\sum x)^2 / n}{n-1}}$, where n is the sample size.

Do not write in this margin.

All questions should be attempted

Marks

1. Calculate $2\frac{1}{6} \times \frac{2}{5}$ **2**

2. Find the gradient of the line which has equation

$$5x + 7y + 35 = 0$$ **2**

3. Change the subject of this formula to 'r' **2**

$$V = \pi r^2 h$$

4. **(a)** Factorise $25x^2 - 49$ **2**

(b) Factorise fully $10x^2 + 9x - 7$ **2**

(c) Simplify $\dfrac{25x^2 - 49}{10x^2 + 9x - 7}$ **1**

Do not write in this margin.

Marks

5. Vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ have components as follows:

$$\boldsymbol{a} = \begin{pmatrix} 4 \\ -3 \\ 4 \end{pmatrix} \quad \text{and} \quad \boldsymbol{b} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

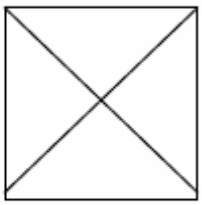
(a) Find the components of the vector represented by $\boldsymbol{a} - 2\boldsymbol{b}$. **1**

(b) Calculate the magnitude of the vector represented by $\boldsymbol{a} - 2\boldsymbol{b}$. **2**

6. Multiply the brackets and simplify $(x-2)(5x^2-4x-2)$ **3**

7.

A polygon with x sides has $\frac{1}{2}x(x-3)$ diagonals.

e.g. 

The square shown has 4 sides
and
$\frac{1}{2}(4)(4-3) = 2$ diagonals.

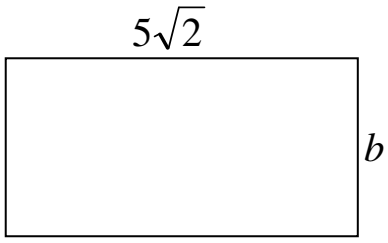
If a polygon has 54 diagonals, how many sides does it have?

4

8. The area of this rectangle is 24 cm². It has length $5\sqrt{2}$.

Calculate its breadth, b, leaving your answer as a surd in its simplest form with a rational denominator.

Dimensions are in centimetres. **3**



Do not write in this margin.

Marks

9. Write as a single fraction in its simplest terms

$$\frac{2}{x-2} - \frac{5}{x+1} \qquad x \neq 2;\ x \neq -1$$ **3**

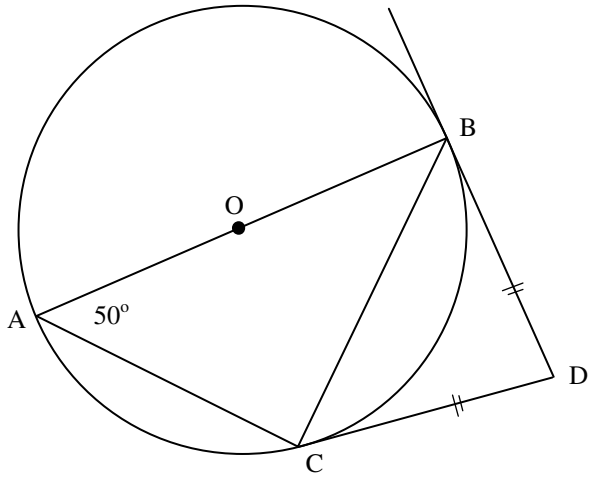
10. Simplify

$x^4 y^3 \div x^6 y$ expressing your answer with positive indices. **2**

11. AB is a diameter and O is the centre of the circle shown below.

BD is a tangent to the circle with B the point of contact.

Triangle BCD is isosceles.

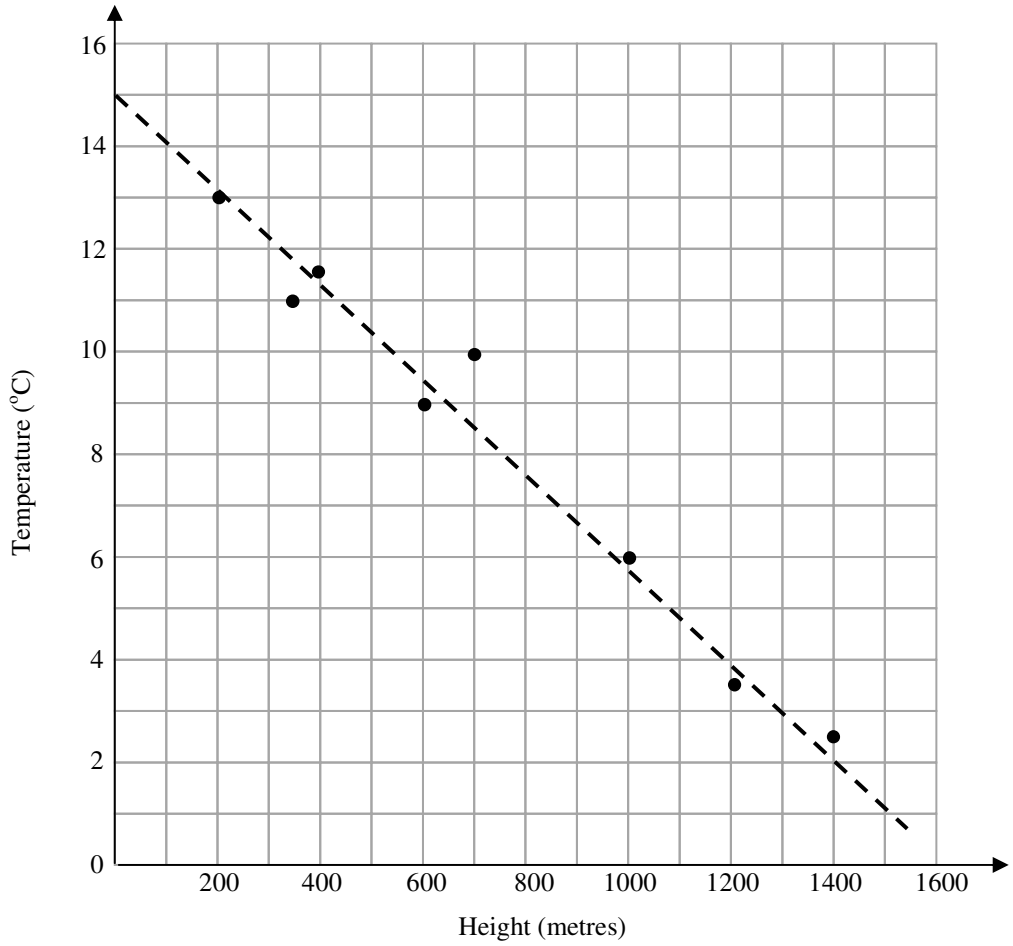


Given that angle BAC = 50°, find the size of angle BDC. **3**

Do not write in this margin.

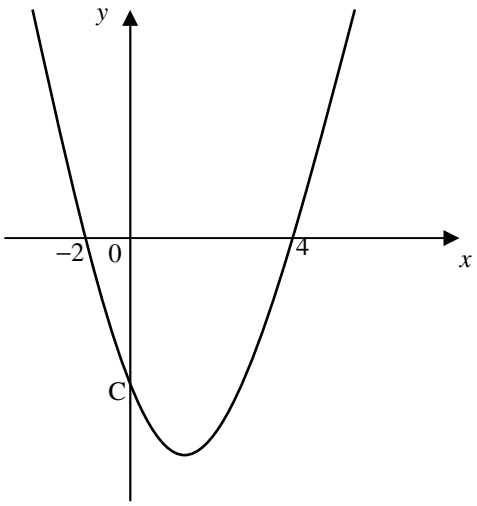
12. The graph shows the height above sea – level, in metres, of eight places in Scotland and the corresponding mean temperature in degrees Celsius. *Marks*

The line of best fit has also been drawn on the graph.



Determine the equation of this line of best fit. **3**

13. The graph shown has equation $y = (x + 2)(x - 4)$.



(a) Find the coordinates of point C, where the graph cuts the y-axis. **2**

(b) Find the coordinates of the turning point. **2**

(c) State the equation of the axis of symmetry of the parabola. **1**

End of Question Paper

Qn	Give one mark for each •	Illustrations for awarding mark
1	**ans:** $\frac{13}{15}$ **2 marks** •[1] changes to improper fraction and simplifies •[2] multiplies	•[1] $\frac{13}{6}\times\frac{2}{5}$ •[2] $\frac{13}{15}$ Note: Candidates may multiply then simplify.
2	**ans:** $m=-\frac{5}{7}$ **2 marks** •[1] rearranges equation to $y = mx + c$ •[2] states gradient of line	•[1] $y = -\frac{5}{7}x-5$ •[2] $m=-\frac{5}{7}$
3	**ans:** $r=\sqrt{\frac{V}{\pi h}}$ **2 marks** •[1] divides both sides by πh •[2] takes square root of both sides	•[1] $r^2=\frac{V}{\pi h}$ •[2] $r=\sqrt{\frac{V}{\pi h}}$
4a	**ans:** $(5x-7)(5x+7)$ **2 marks** •[1] one bracket correct •[2] second bracket correct	•[1] $(5x-7)\ldots\ldots$ •[2] $\ldots..(5x+7)$
b	**ans:** $(5x+7)(2x-1)$ **2 marks** •[1] one bracket correct •[2] second bracket correct	•[1] $(5x+7)(\ldots..$ •[2] $\ldots.(2x-1)$
c	**ans:** $\frac{5x-7}{(2x-1)}$ **1 mark** •[1] simplifies	•[1] $\frac{(5x-7)}{(2x-1)}$
5a	**ans:** $\begin{pmatrix}2\\-3\\6\end{pmatrix}$ **1 mark** •[1] states components	•[1] $\begin{pmatrix}2\\-3\\6\end{pmatrix}$
b	**ans:** 7 **2 marks** •[1] knows how to find magnitude •[2] evaluates	•[1] $\sqrt{2^2+(-3)^2+6^2}$ •[2] $\sqrt{49}=7$

Qn	Give one mark for each •	Illustrations for awarding mark
6	**ans: $5x^3 - 14x^2 + 6x + 4$** **3 marks** •1 starts to multiply •2 completes multiplication •3 simplifies	 •1 $5x^3 - 4x^2 - 2x$……. •2 $-10x^2 + 8x + 4$ •3 $5x^3 - 14x^2 + 6x + 4$ [must have x^3 term]
7	**ans: 12 (Source: Pegasys Credit prelim 200…)** **4 marks** •1 sets up equation •2 equates to zero •3 factorises correctly •4 selects correct answer **N.B.** *AWARD 2/4 for '12' with no working and AWARD 4/4 for '12' with evidence of trial and improvement.*	 •1 $\frac{1}{2}x(x-3) = 54$ •2 $x^2 - 3x - 108 = 0$ •3 $(x-12)(x+9) = 0$ •4 $x = 12$
8	**ans: $\frac{12\sqrt{2}}{5}$** **3 marks** •1 knows how to find breadth •2 knows how to rationalize denominator •3 simplifies	 •1 $24 \div 5\sqrt{2}$ •2 $\times \sqrt{2} / \sqrt{2}$ •3 $(12\sqrt{2}) / 5$
9	**ans: $\frac{12 - 3x}{(x-2)(x+1)}$** **3 marks** •1 correct denominator •2 correct numerator •3 simplifies numerator	 •1 $(x-2)(x+1)$ •2 $2(x+1) - 5(x-2)$ •3 $12 - 3x$
10	**ans: $\frac{y^2}{x^2}$** **2 marks** •1 starts to simplify •2 writes with positive powers	 •1 $y^2 x^{-2}$ •2 $\frac{y^2}{x^2}$
11	**ans: $\angle BDC = 80^o$** **3 marks** •1 realises $\angle ACB$ is right and finds $\angle ABC$ •2 finds $\angle CBD$ and $\angle BCD$ •3 finds $\angle BDC$	 •1 $\angle ACB = 90^o$; $\angle ABC = 40^o$ •2 $\angle CBD = 50^o$; $\angle BCD = 50^o$ •3 $\angle BDC = 80^o$ Angle must be stated explicitly
12	**ans: $T = -1/100H + 15$** **3 marks** •1 finds gradient •2 identifies y – intercept •3 states equation	 •1 $m = -1/100$ •2 $c = 15$ •3 $T = -1/100H + 15$ [or equivalent]

Qn	Give one mark for each •	Illustrations for awarding mark
13a	**ans: C(0, – 8) 2 marks**	
	•[1] knows to sub $x = 0$ •[2] states point C	•[1] $y = (0 + 2)(0 – 4)$ •[2] C(0, – 8)
b	**ans: (1, – 9) 2 marks**	
	•[1] knows to substitute $x = 1$ into equation •[2] states coordinates of T.P.	•[1] $y = (1 + 2)(1 – 4)$ •[2] (1, – 9)
c	**ans: $x = 1$ 1 mark**	
	•[1] states equation of axis of symmetry	•[1] $x = 1$
		Total 40 marks

N5

Elgin Academy

Prelim Examination 2013 / 14

MATHEMATICS

National Qualifications - National 5

Paper 2 (Calculator)

Covering all three Units

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Read carefully what is printed below

Total marks - 50

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2. Use **blue** or **black** ink. Pencil may be used for graphs and diagrams only.
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Do not write in this margin.

Marks

All questions should be attempted

1. Jasper invests £10500 in a bank that pays 3·6% interest per annum.

If Jasper does not withdraw any money, how much will he have in his account after 3 years? **3**

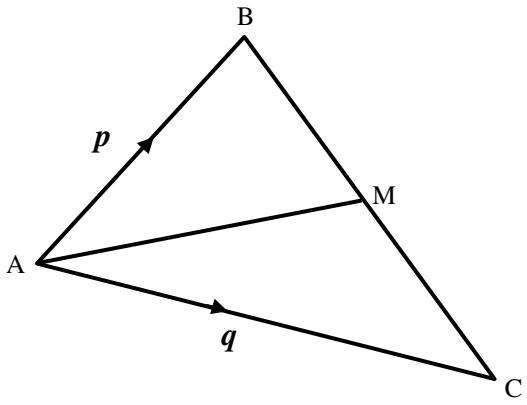
2. 46 795 runners took part in the 2011 New York marathon which set a new world record for the number of runners finishing a marathon.

During the race each runner was supplied with 2·5 litres of water.

How many litres of water were supplied altogether?

Give your answer in Scientific Notation correct to 3 significant figures. **2**

3. In the diagram, M is the mid – point of BC. $\overrightarrow{AB}$ represents vector $\boldsymbol{p}$ and $\overrightarrow{AC}$ represents vector $\boldsymbol{q}$.



(a) Express $\overrightarrow{BC}$ in terms of $\boldsymbol{p}$ and $\boldsymbol{q}$. **2**

(b) Express $\overrightarrow{BM}$ in terms of $\boldsymbol{p}$ and $\boldsymbol{q}$. **1**

Marks

4. Solve the quadratic equation $2x^2 - 3x - 4 = 0$

giving your answers correct to one decimal place. **4**

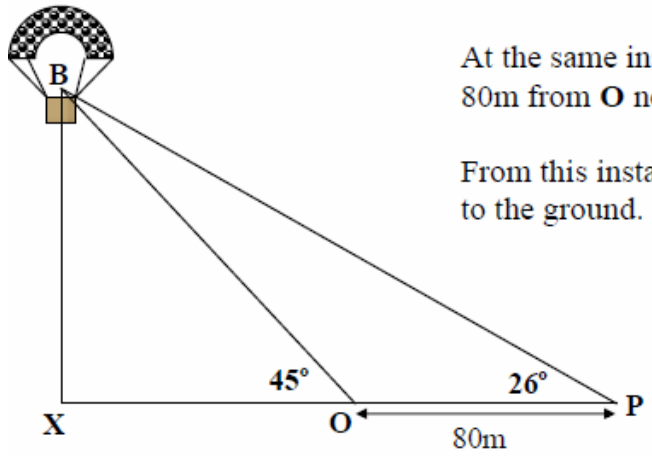
5.

A basket, **B**, containing medical supplies is descending vertically at a constant speed over a point **X**.

An observer, **O**, notes that at a certain instant the angle of elevation to **B** is 45°.

At the same instant a second observer, **P**, standing 80m from **O** notes the angle of elevation to **B** is 26°.

From this instant the basket takes 6 minutes to fall to the ground.



Calculate the speed at which the basket falls to the ground in m/min.

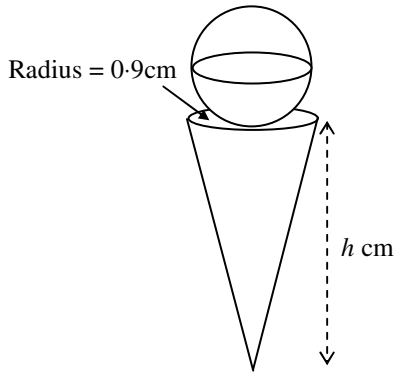
6

6. I bought an antique painting a few years ago. It has gained 45% in value since it was bought and is now worth £6525.

Calculate how much the painting was worth when it was bought. **3**

7. A metal bottle stopper is made up from a cone topped with a sphere.

The sphere has diameter 1·5cm.



Marks

(a) Calculate the volume of the sphere. **2**

The total volume of the stopper is 6 cm³

(b) If the cone has radius 0·9cm, calculate the height, h cm, of the cone. **3**

8. Last year, for a Mathematical Competition the organisers bought 30 medals and 4 trophies for £37.50

(a) Write down an equation in m and t to illustrate this information. **1**

This year they bought in 50 medals and 8 trophies which cost them £66.50.

(b) Form another equation in m and t to illustrate this information. **1**

(c) If the cost of a medal and a trophy remained the same for both years, find **algebraically** the cost of 1 medal and 1 trophy. **4**

Do not write in this margin.

Marks

9. Over the course of six weeks couple A scored the following marks out of 40 in a dancing competition:

33 30 31 26 36 36

(a) Calculate the mean and standard deviation of these scores. **4**

Over the same six weeks, couple B's scores were, on average, the same as couple A but were slightly less consistent.

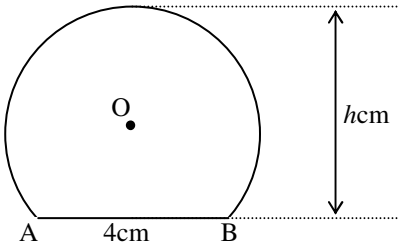
(b) Write down a possible standard deviation for couple B's scores. **1**

10. The diagram shows the cross section of a paper weight.
It consists of part of a circle with a horizontal base.

The centre of the circle is O and it has radius 5cm.

AB is a chord of the circle and measures 4cm.

Calculate the height, *h*cm, of the paperweight. **4**

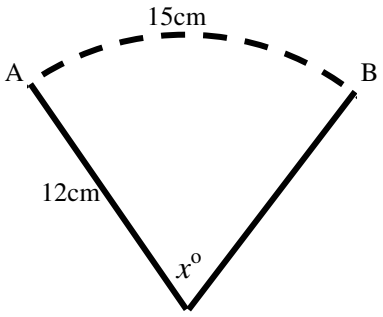


Marks

11. A metronome is a music tool which helps players with rhythm and tempo.

A weight on the pendulum is adjusted so that the metronome swings back and forth to give the correct tempo for a piece of music.

For one particular piece the pendulum is set to a length of 12cm and as it swings it traces out an arc of a circle, AB, of length 15cm.



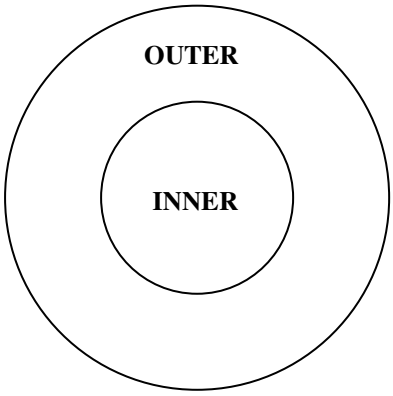
Calculate, to the nearest degree, the angle x^o, through which the pendulum swings. **3**

Marks

12.

Chris threw 100 darts at a target and a statistician recorded his results as follows:

- **1 out of 5** landed in the inner section
- **17 out of 25** landed in the outer section
- **4 out of 50** missed the target completely.



Did the statistician record the results correctly?
Give a reason for your answer. **3**

13. These two tubes of toothpaste are mathematically similar. The cost of the tube depends on its volume.

The larger tube is 18cm long and holds 240 ml of toothpaste, the smaller one measures 12 cm.

18cm

12cm

If the larger tube costs £1.44, how much should the small one cost? **3**

End of Question Paper

Qn	Give one mark for each •	Illustrations for awarding mark
1	**ans: £11675** **3 marks** •1 correct multiplier •2 correct method •3 answer	 •1 $1{\cdot}036$ •2 $10500 \times 1{\cdot}036^3$ •3 £11675
2	**ans : $1{\cdot}17 \times 10^5$** **2 marks** •1 knows to multiply •2 answer in Scient. Not. correctly rounded	 •1 $46\,796 \times 2{\cdot}5 = 116\,987{\cdot}5$ •2 $1{\cdot}17 \times 10^5$
3a	**ans : $q - p$** **2 marks** •1 chooses correct path •2 answer	 •1 $\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$ •2 $q - p$
b	**ans: ½ $(q - p)$** **1 mark** •1 answer	 •1 ½ $(q - p)$
4	**ans : 2·4, –0·9** **4 marks** •1 knows to use quadratic formula •2 calculates $b^2 - 4ac$ •3 subs correctly into formula •4 states both roots correctly rounded	 •1 evidence •2 41 •3 $\frac{3 \pm \sqrt{41}}{2 \times 2}$ •4 2·4, –0·9
5	**ans : 12.7m/min** **(Source Pegasys Credit prelim 2007)** **6 marks** •1 finds the missing angle required •2 sets up sine rule calculation correctly •3 processes sine rule calculation •4 uses result from •3 in subsequent calc •5 processes calculation correctly •6 uses s = d/t to find speed	 •1 19° •2 $\frac{x}{\sin 135} = \frac{80}{\sin 19}$ •3 173.75m •4 $\sin 26 = \frac{h}{173.75}$ •5 h = 76.2 m •6 speed = 76.2/6 = 12.7m/min
6	**ans : £4500** **3 marks** •1 knows that 145% = £6525 •2 knows to divide £6525 by 1·45 •3 answer	 •1 145% = £6525 •2 100% = £6525 ÷ 1·45 •3 £4500
7a	**ans: 1·8 cm³** **2 marks** •1 substitutes values in appropriate formula •2 answer **ans: 5 cm** **3 marks**	 •1 $\frac{4}{3} \times \pi \times 0{\cdot}75^3$ •2 1·8 m³ [accept any rounding]
b	•1 finds volume of cone •2 substitutes into appropriate formula •3 finds height of cone	•1 $6 - 1{\cdot}8 = 4{\cdot}2\text{m}^3$ •2 $4{\cdot}2 = \frac{1}{3} \times \pi \times 0{\cdot}9^2 \times h$ •3 $h = 5\text{cm}$

Qn	Give one mark for each •	Illustrations for awarding mark
8a	**ans: $30m + 4t = 37{\cdot}5$ 1 mark**	
	•[1] constructs equation	•[1] $30m + 4t = 37{\cdot}5$
b	**ans: $50m + 8t = 66{\cdot}5$ 1 mark**	
	•[1] constructs equation	•[1] $50m + 8t = 66{\cdot}5$
c	**ans: medal costs £0.85; trophy costs £3 4 marks**	
	•[1] scales equations •[2] finds value for 'm' •[3] finds value for 't' •[4] communicates answer	•[1] evidence •[2] $m = 0{\cdot}85$ •[3] $t = 3$ •[4] medal costs £0.85; trophy costs £3
9a	**ans: mean = 32; S.D. = 3·8 4 marks**	
	•[1] finds mean •[2] finds $(\sum x)^2$ and $\sum x^2$ •[3] substitutes into formula •[4] answer	•[1] 192/6 = 32 •[2] 36864; 6218 •[3] $\sqrt{\frac{6218 - \left(\frac{36864}{6}\right)}{5}}$ •[4] 3·8 [accept any correct rounding]
	Or	
	•[1] finds mean •[2] finds deviations squared •[3] knows how to find SD •[4] answer	•[1] 192/6 = 32 •[2] 1 + 4 + 1 + 36 + 16 + 16 = 74 •[3] $\sqrt{\frac{74}{5}}$ •[4] 3·8 [accept any correct rounding]
b	**ans: value > 3·8 1 mark**	
	•[1] states possible value for SD	•[1] SD > 3·8
10	**ans: 9.6m 4 marks**	
	•[1] assembles facts in right-angled triangle •[2] knows to use Pythagoras' •[3] answer •[4] finds height of paperweight	•[1] evidence of R.A.T. •[2] $5^2 - 2^2$ •[3] $\sqrt{21} = 4.6$m •[4] 4·6 + 5 = 9.6m
11	**ans : 72^o 3 marks**	
	•[1] uses appropriate ratios •[2] substitutes and re-arranges •[3] answer	•[1] $x/360 = 15/\pi D$ •[2] $x/360 = 15/(\pi \times 24)$; $x = (15 \times 360) / 24\pi$ •[3] 72^o

Qn	Give one mark for each •	Illustrations for awarding mark
12	**ans :** ***No, there are 4 unrecorded*** **3 marks** **(Source: dept staff)** •[1] denominator of 100 or % •[2] finds total correctly by addition •[3] conclusion	 •[1] 20%, 68%, 8% •[2] 96% •[3] No, there are 4 unrecorded
13	**ans: 43p** **3 marks** •[1] finds reduction scale factor •[2] finds volume scale factor •[3] multiplies by VSF to answer	 •[1] 12/18 = 2/3 •[2] $(2/3)^3 = 8/27$ •[3] 8/27 × £1.44 = 43p
		Total 50 marks