

Solutions

12 Functions

Properties of the Parabola

1. a) $a = -1, b = 3$ (roots of the equation)
b) The point $(0, -6)$ lies on the curve, so it will satisfy the equation of the curve.
Hence, $-6 = k(0 + 1)(0 - 3)$ so, $-6 = -3k$
 $k = 2$
c) Axis of symmetry is $x = 1$.
When $x = 1, y = 2(1 + 1)(1 - 3)$
Hence $y = -8$
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2. a) Cuts y axis when $x = 0$, so $y = -12$
b) B and C are roots of eqn. $x^2 + x - 12 = 0$
factorise $(x + 4)(x - 3) = 0$ hence $x = -4$, or 3
B is $(4, 0)$ and C is $(3, 0)$
c) Axis of symmetry is $x = -\frac{1}{2}$.
When $x = -\frac{1}{2}, y = \left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right) - 12$
Hence $y = \frac{1}{4} - \frac{1}{2} - 12 \rightarrow -12\frac{1}{4}$
Co-ords of min t.p. are $(-\frac{1}{2}, -12\frac{1}{4})$
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3. Use the formula with $a = 3, b = 7, c = -2$
 $x = -0.21$ or $x = -2.12$
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4. a) When $x = 0, y = -3$
b) Solve the equation by factorisation
 $4x^2 + 4x - 3 = 0 \quad (2x - 1)(2x + 3) = 0$
hence $x = \frac{1}{2}$ or $x = -\frac{3}{2}$
c) axis of symmetry is $x = -\frac{1}{2}$
when $x = -\frac{1}{2} y = -4$ co-ords of min t.p. $(-\frac{1}{2}, -4)$
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Applications of the parabola

1. a) Area of glass = $(7 - 2x)(10 - 2x)$
 $A = 70 - 14x - 20x + 4x^2$ Hence, $A = 4x^2 - 34x + 70$
b) $28 = 4x^2 - 34x + 70$
Re-arrange: $4x^2 - 34x + 42 = 0$
Divide by 2: $2x^2 - 17x + 21 = 0$
Factorise: $(2x - 3)(x - 7) = 0$
Hence $x = 1\frac{1}{2}$ or $x = 7$
 x cannot be 7, since this is width of frame,
So $x = 1\frac{1}{2}$ cms
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2. a) Area of A is: $(x + 6)(x - 1)$
Area of B is: $3(x + 3)$
b) So, $(x + 6)(x - 1) = 3(x + 3)$
Hence, $x^2 + 6x - x - 6 = 3x + 9$
simplify: $x^2 + 2x - 15 = 0$
factorise: $(x + 5)(x - 3) = 0$
so, $x = 3$ or $x = -5$. $x = -5$ is not possible
Hence $x = 3$
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3. a) Solve the quadratic by factorisation
 $8 + 2x - x^2 = 0$ Hence, $(4 - x)(2 + x) = 0$
 $x = 4$ or $x = -2$, so F is $(4, 0)$
The fly is 4 feet to the right of the snake.
b) axis of symmetry is when $x = 1$
Hence max height is $H = 8 + 2 - 1 = 9$ feet.
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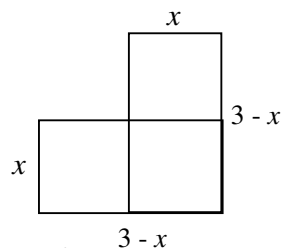
4. $H(3) = 9 + 6(3) - 3(3)^2 = 0$

This indicates that the shell is now level with the cliff again.

5. a) Since $BC = CD$ then

$$2BC + 2x = 6 \rightarrow BC + x = 3 \quad \text{So, } BC = 3 - x$$

- b) Area of rectangle
 $= x(3 - x)$
There are 2 rectangles
but then we have counted
the square twice.



$$\text{Hence Area} = x(3 - x) + x(3 - x) - x^2$$

$$A = 3x - x^2 + 3x - x^2 - x^2$$

$$A = 6x - 3x^2$$

- c) Find the roots of the equation $6x - 3x^2 = 0$
Factorise: $3x(2 - x) = 0$
Hence $x = 0$ or $x = 2$ $x = 0$ is not possible, So $x = 2$.
Axis of symmetry is $x = 1$
Max value is on axis of symmetry: $A = 6 - 3 = 3 \text{ m}^2$
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6. a) $l = w + 2$
b) Area of extension is: $w(w + 2) \rightarrow w^2 + 2w$
This must not be more than 40% original size
 $120 \times 0.4 = 48$ So $w^2 + 2w = 48$ (largest extension)
Hence $w^2 + 2w - 48 = 0$ so $(w - 6)(w + 8) = 0$
 $w = 6$ or -8 (not possible) Width = 6, Length = 8
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7. a) $18 - 2x$ cms
b) $V = x(18 - 2x) \times 100 \quad V = 1800x - 200x^2$
c) Put $1800x - 200x^2 = 0$ and solve equation by
factorising: $200x(9 - x) = 0 \quad x = 0$ or $x = 9$
maximum is on axis of symmetry $x = 4\frac{1}{2}$
dimensions of gutter are 9 cm wide \times 4 $\frac{1}{2}$ cm high
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