

## Solutions

### 13 Making & Using Formulae

1.  $DB = 2x$  Let  $TD = h$  cms  
 Area TDB =  $\frac{1}{2}$  base  $\times$  height =  $\frac{1}{2} \times h \times 2x = hx$   
 Area of clipboard =  $3x \times 4x = 12x^2$   
 Area triangle =  $\frac{1}{4}$  area clipboard  
 $hx = 3x^2$  so,  $h = 3x$

2. Put  $d = 20$  into formula

$$20 = \frac{n(n-3)}{2} \rightarrow 40 = n^2 - 3n$$

Re-arrange:  $n^2 - 3n - 40 = 0$  factorise to solve  
 $(n-8)(n+5) = 0$  so  $n = 8$  or  $n = -5$   
 number of sides must be 8 (-5 not possible)

3. a)  $3 \times 25 + 5 \times 3 = 90$  pence  
 b)  $75 + (m-3) \times 5 \rightarrow 75 + 5m - 15 \rightarrow 60 + 5m$   
 c)  $80 + (m-2) \times 2 \rightarrow 80 + 2m - 4 \rightarrow 76 + 2m$   
 $76 + 2m < 60 + 5m$   
 $16 < 3m$   $m > 5.33$  mins  
 Minimum number of minutes = 6 minutes

4. a) put  $c = 3$   $I = \frac{20}{2^3} \rightarrow \frac{20}{8} = 2.5$   
 b) put  $I = 10$   
 $10 = \frac{20}{2^c} \rightarrow 2^c = \frac{20}{10} \rightarrow 2^c = 2$  So,  $c = 1$   
 c) max intensity  $2^c$  is smallest i.e. when  $c = 0$   
 then  $2^0 = 1$  max intensity is 20

5. a)  $30 + x$   
 b) Area =  $(30 + x)(20 + x) \rightarrow 600 + 30x + 20x + x^2$   
**Area =  $600 + 50x + x^2$**   
 c) New Area =  $30 \times 20 \times 1.4 = 840$   
 Solve equation:  $840 = 600 + 50x + x^2$   
 re-arrange:  $x^2 + 50x - 240 = 0$   
 Use formula with  $a = 1$ ,  $b = 50$   $c = -240$   
 $x = 4.41$  cm or  $x = -54.41$  cms  
 Hence min dimensions are: 35 cms by 25 cms  
 (nearest cm - remember dimensions are minimum)

6. Volume of space = Vol. Cylinder - Vol cuboid  
 Calculate volume of cylinder.

$$V = \pi r^2 h \rightarrow V = \pi \times 6^2 \times 20 = 720\pi \text{ cm}^3$$

Cross section of cuboid is a square

Diagonal = 12 cm, Area =  $\frac{1}{2}$  diag  $\times$  diag  
 Area =  $\frac{1}{2} \times 12 \times 12 = 72 \text{ cm}^2$

Or use Pythagoras

$$\text{Side of square} = \sqrt{6^2 + 6^2} = \sqrt{72} \text{ cm}$$

$$\text{Area of square} = \sqrt{72} \times \sqrt{72} = 72 \text{ cm}^2$$

Volume of cuboid =  $72 \times 20 = 1440$

Hence vol of space =  $720\pi - 1440 = 720(\pi - 2)$

7. a) Base rate = £425 per person  
 2 extra adults so less £ 60 per person  
 Hence cost :  $365 \times 4 = \text{£ } 1460$   
 b) Base rate = £425 per person for P persons  
 (P - 2) extra adults,  
 so reduction is: £ 30  $\times$  (P - 2) per person  
 Hence cost :  $425 - 30(P - 2)$  per person  
 For P persons: Cost = £ P[425 - 30(P - 2)]

8. Supplement of y is  $180^\circ - y$   
 Angles in triangle add up to  $180^\circ$   
 So,  $a + b + 180 - y = 180$   
 Hence,  $y = a + b$

9. a)  $(42 - 15)$  charged at 35p per min = £9.45  
 = rental £ 17.50, Total cost = £ 26.95  
 b)  $(t - 15) \times 0.35$  for calls + rental of £17.50  
 $\rightarrow 0.35t - 5.25 + 17.5 \rightarrow \text{£ } 12.25 + 0.35t$

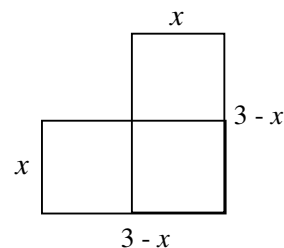
10. a) Since  $BC = CD$  then

$$2BC + 2x = 6 \rightarrow BC + x = 3 \text{ So, } BC = 3 - x$$

- b) Area of rectangle

$$= x(3 - x)$$

There are 2 rectangles  
 but then we have counted  
 the square twice.



$$\text{Hence Area} = x(3 - x) + x(3 - x) - x^2$$

$$A = 3x - x^2 + 3x - x^2 - x^2$$

$$A = 6x - 3x^2$$

- c) Find the roots of the equation  $6x - 3x^2 = 0$   
 Factorise:  $3x(2 - x) = 0$   
 Hence  $x = 0$  or  $x = 2$   $x = 0$  is not possible, So  $x = 2$ .  
 Axis of symmetry is  $x = 1$   
 Max value is on axis of symmetry:  $A = 6 - 3 = 3 \text{ m}^2$

11. a) Pupils  $12 \times \text{£}4.50 = \text{£}54$   
 Adult: 1 free so  $2 \times \text{£}7.00 = \text{£}14.00$   
 Total cost = £68.00  
 b) Cost of pupils: £  $4 \times p$   
 Adults: 2 free  $d - 2$  adults pay £  $6(d - 2)$   
 Total cost: £  $4p + 6(d - 2)$

12. Substitute  $N = 26$  into formula

$$26 = \frac{30v}{2 + v} \text{ re-arrange } 26(2 + v) = 30v$$

$$\text{Hence, } 52 + 26v = 30v \rightarrow 4v = 52 \quad v = 13$$

Speed of cars = 13 metres per second.

13. a)  $C = 15d$   
 b)  $C = 50 + 10d$   
 c) Eurocar:  $170 = 15d$   $d = 170 \div 15$   $d = 11.333\dots$   
 Apex:  $170 = 50 + 10d$   $10d = 120$   $d = 12$   
 Apex will give them 12 days.  
 (NB could also hire car for 3 days from Eurocar with deposit)

## Solutions

### 13 Making & Using Formulae (continued)

$$14. \quad s = \frac{8.6 + 7.4 + 10 + 9.1}{2} = 17.55$$

$$A = \sqrt{(17.55 - 8.6)(17.55 - 7.4)(17.55 - 10)(17.55 - 9.1)}$$

$$A = \sqrt{8.95 \times 10.15 \times 7.55 \times 8.45} = \sqrt{5795.524} = 76.128..$$

$$A = 76 \text{ cm}^2 \text{ (2 sf)}$$

$$15. \quad \text{a) Expenses: } 250 \times 0.29 + 300 \times 0.15 = \text{£}117.50$$

$$\text{b) } E = 250 \times 0.29 + (t - 250) \times 0.15$$

$$E = 72.50 + 0.15t - 37.50 = 35 + 0.15t$$

$$16. \quad \text{a) } 16$$

$$\text{b) i) } 1300 \div 150 = 8.66.. \text{ Integral part} = 8$$

$$\text{ii) } \left[ \frac{1300}{B} \right] \times \left[ \frac{1000}{L} \right]$$

$$17. \quad \text{a) } 18 - 2x \text{ cms}$$

$$\text{b) } V = x(18 - 2x) \times 100 \quad V = 1800x - 200x^2$$

$$\text{c) Put } 1800x - 200x^2 = 0 \text{ and solve equation by}$$

$$\text{factorising: } 200x(9 - x) = 0 \quad x = 0 \text{ or } x = 9$$

$$\text{maximum is on axis of symmetry } x = 4 \frac{1}{2}$$

dimensions of gutter are 9 cm wide  $\times$  4  $\frac{1}{2}$  cm high

$$18. \quad \text{a) Cost} = \text{£}13.50 + \text{£}0.75 \times 4 = \text{£} 16.50$$

$$\text{b) } C = 13.50 + 0.75 \times (w - 10)$$

$$C = 13.50 + 0.75w - 7.5 = 6 + 0.75w$$

$$19. \quad \text{Ellipse will cut x-axis at } -6 \text{ and } 6$$

$$\text{and y-axis at } 4 \text{ and } -4$$

(Look for the patterns in the formulae

– denominators are squares of where it cuts the axes.)

$$20. \quad \text{a) Use formula}$$

$$P = \frac{(40+15)(40-15+1)}{2} \rightarrow \frac{55 \times 26}{2} \rightarrow 715$$

$$\text{b) There are } a \text{ on the top row and } 2a \text{ on the bottom}$$

row, so put  $b = 2a$  in formula

$$P = \frac{(2a+a)(2a-a+1)}{2} \rightarrow \frac{(3a)(a+1)}{2} \rightarrow \frac{3a^2+3a}{2}$$

$$\text{c) Can } P = 975 \text{ with whole number solutions?}$$

$$975 = \frac{3a^2+3a}{2} \rightarrow 1950 = 3a^2+3a$$

$$3a^2+3a-1950=0 \text{ dividing by } 3$$

$$a^2+a-630=0$$

Try solving with the formula

with  $a = 1$ ,  $b = 1$ ,  $c = -630$

find that solution involves  $\sqrt{2521} = 50.2..$

So no whole number solutions

$$21. \quad \text{a) angle above } b \text{ is } 72^\circ \text{ (corresponding)}$$

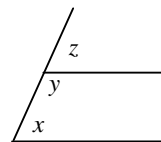
$$\text{hence } b = 180 - 72 = 108^\circ$$

$$\text{b) In the diagram}$$

$$\angle x = \angle z \text{ (corresponding)}$$

$$\angle y + \angle z = 180^\circ \text{ (supplementary)}$$

$$\text{hence } \angle y + \angle x = 180^\circ$$



$$22. \quad \text{a) Adults: } \text{£}555 \text{ Child: FREE Extra nights: } \text{£}29 \times 3$$

$$\text{Total cost} = \text{£} 642$$

$$\text{b) } C = 555 + 29(t - 14) \rightarrow C = 555 + 29t - 406$$

$$C = 149 + 29t$$

$$23. \quad \text{a) Area of border} = x^2 - y^2 = 48$$

$$\text{Hence } (x - y)(x + y) = 48$$

$$\text{b) Factors of } 48 \text{ are}$$

$$48 \times 1, 24 \times 2, 16 \times 3, 12 \times 4, 8 \times 6$$

Since  $x$  and  $y$  are greater than 10, then  $x + y > 20$

so only need to consider  $48 \times 1$  and  $24 \times 2$

$$\text{hence } x + y = 48 \text{ and } x - y = 1$$

no solution – numbers need to differ by 1

so one must be odd, and 48 is even

$$\text{or } x + y = 24 \text{ and } x - y = 2$$

$$\text{so, } x = 13, y = 11$$

$$\text{Hence: } x = 13, y = 11$$

$$24. \quad \text{a) If } AB = 2 \text{ then } BC = 2 \text{ (it is a square)}$$

$$\text{By Pythagoras: } AC^2 = 2^2 + 2^2 \text{ so } AC = \sqrt{8} \rightarrow 2\sqrt{2}$$

$$\text{b) In any square of side } a.$$

$$\text{Diagonal} = \sqrt{a^2 + a^2} = \sqrt{2a^2} = a\sqrt{2}$$

$$\text{Ratio of side to diagonal is: } a : a\sqrt{2}$$

$$\text{which is } 1 : \sqrt{2}$$

$$25. \quad \text{a) i) } 12h \text{ minutes}$$

$$\text{ii) } \frac{v}{100} \times 10 \rightarrow \frac{v}{10} \text{ minutes}$$

$$\text{iii) } \frac{12h}{60} + \frac{v}{600} \rightarrow \frac{120h}{600} + \frac{v}{600} \rightarrow T = \frac{120h+v}{600}$$

$$\text{b) } T = \frac{120 \times 8 + 900}{600} \rightarrow \frac{1860}{600} \rightarrow 3.1 \text{ hours}$$

$$\text{Total time} = 3.1 \text{ hours each way} = 6.2 \text{ hours}$$

No, it should not be started

Since 1300 to 1900 is only 6 hours.

$$27. \quad \text{a) } \text{£}5 + \text{£}26 + \text{£}18 = \text{£}49$$

$$\text{b) Total Fee payable} = 5 + 26 + (P - 15)$$

$$= 31 + P - 15 = 16 + P$$

$$26. \quad \text{a) Plot graph when } t = 0, M = 80 \text{ (0, 80)}$$

$$(1, 40), (2, 20), (3, 10), (4, 5)$$

$$\text{b) } \frac{5}{8} = 80(2)^{-t} \rightarrow \frac{5}{8} = \frac{80}{2^t} \rightarrow 2^t = \frac{640}{5} = 128$$

$$2^t = 2^7 \text{ hence } t = 7. \text{ It will take } 7 \text{ years.}$$