

Solutions

14 Trig Graphs & Equations

1. hypotenuse = 5 (Pythagoras – or 3,4,5 triangle)

$$\sin x = \frac{3}{5} \quad \cos x = \frac{4}{5}$$

$$\sin^2 x + \cos^2 x = \left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2 = \frac{9}{25} + \frac{16}{25} = \frac{25}{25} = 1$$

2. a is the amplitude, using symmetry, top of wave is 5
Hence $a = -5$ (since sine wave is inverted)

b is number of waves in 360°

whole wave will take up 120°

so $b = 3$

$$a = -5, \quad b = 3$$

3. a) 1.30 pm is 1.5 hours after midnight,
put $t = 1.5$ into the formula

$$D = 12.5 + 9.5 \sin(30 \times 1.5)$$

$$\text{Depth} = 19.217\dots = 19.2 \text{ metres (1 dp)}$$

- b) Maximum depth is when sine is maximum ($= 1$)
Max depth = $12.5 + 9.5 = 22$ metres

Minimum depth is when sine is minimum ($= -1$)

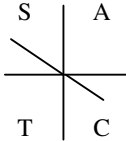
$$\text{Min depth} = 12.5 - 9.5 = 3 \text{ metres}$$

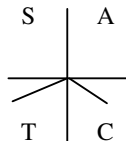
$$\text{Maximum difference is } 22 - 3 = 19 \text{ metres.}$$

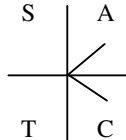
4. $y = k \sin ax^\circ$ $k = 3$ (amplitude)
 $a = 2$ (number of waves in 360°)

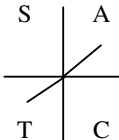
5. $y = a \cos bx^\circ$ $a = 3$ (amplitude)
 $b = 2$ (number of waves in 360°)

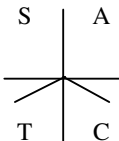
Solving Equations

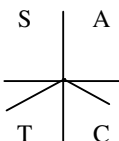
1. $3 \tan x + 5 = 0 \rightarrow \tan x = -\frac{5}{3}$ 
- $x = \tan^{-1} \frac{5}{3}$ acute $x = 59.03\dots^\circ$
- $x = 180 - 59 = 121^\circ$ or $x = 360 - 59 = 301^\circ$

2. $2 + 3 \sin x = 0 \rightarrow \sin x = -\frac{2}{3}$ 
- $x = \sin^{-1} \frac{2}{3}$ acute $x = 41.81\dots^\circ$
- $x = 180 + 42 = 222^\circ$ or $x = 360 - 42 = 318^\circ$

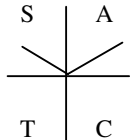
3. $7 \cos x - 2 = 0 \rightarrow \cos x = \frac{2}{7}$ 
- $x = \cos^{-1} \frac{2}{7}$ acute $x = 73.398\dots^\circ$
- $x = 73^\circ$ or $x = 360 - 73 = 287^\circ$

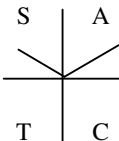
4. $5 \tan x - 9 = 0 \rightarrow \tan x = \frac{9}{5}$ 
- $x = \tan^{-1} \frac{9}{5}$ acute $x = 60.945\dots^\circ$
- $x = 61^\circ$ or $x = 180 + 61 = 241^\circ$

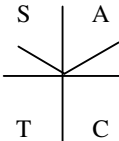
5. $5 \sin x + 2 = 0 \rightarrow \sin x = -\frac{2}{5}$ 
- $x = \sin^{-1} \frac{2}{5}$ acute $x = 23.578\dots^\circ$
- $x = 180 + 24 = 204^\circ$ or $x = 360 - 24 = 336^\circ$

6. $\tan 40 = 2 \sin x + 1 \rightarrow \sin x = -\frac{0.1609}{2}$ 
- $x = \sin^{-1} \frac{0.1609}{2}$ acute $x = 4.614\dots^\circ$
- $x = 180 + 5 = 185^\circ$ or $x = 360 - 5 = 355^\circ$

7. $2 \tan 24 = \tan q \rightarrow \tan q = 0.8905$
- $x = \tan^{-1} 0.8905$ acute $x = 41.683\dots^\circ$
- q is an acute angle, so $q = 42^\circ$ (2 sf)

8. Solve $y = \sin x$ and $y = 0.4 \rightarrow \sin x = 0.4$
- $x = \sin^{-1} 0.4$ acute $x = 23.578\dots^\circ$ 
- $x = 24^\circ$ or $x = 180 - 24 = 156^\circ$
- A is $(24^\circ, 0.4)$ and B is $(156^\circ, 0.4)$

9. a) amplitude = 3, so $a = 3$
since max is at 90° , there is 1 wave in 360°
hence $b = 1$
- b) $3 \sin x = 2 \rightarrow \sin x = \frac{2}{3}$ 
- $x = \sin^{-1} \frac{2}{3}$ acute $x = 41.81\dots^\circ$
- $x = 42^\circ$ or $x = 180 - 42 = 138^\circ$
- P is $(42^\circ, 2)$ and Q is $(138^\circ, 2)$

10. a) S is $(90^\circ, 1)$ 
- b) $\sin x = 0.5$
- $x = \sin^{-1} 0.5$ acute $x = 30^\circ$
- $x = 30^\circ$ or $x = 180 - 30 = 150^\circ$
- T is $(30^\circ, 0.5)$ and P is $(150^\circ, 0.5)$

Solutions

14 Trig Graphs & Equations (continued)

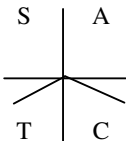
11. a) A is $(90^\circ, 0)$

b) $\cos x = -0.5$

$x = \cos^{-1} 0.5$ acute $x = 60^\circ$

$x = 180 + 60 = 240^\circ$ or $x = 360 - 60 = 300^\circ$

B is $(240^\circ, -0.5)$ and C is $(300^\circ, -0.5)$



12. a) Maximum value of H is when cosine is maximum (= 1)

$h = 1.9 + 0.3 = 2.2$ metres

b) After 8 seconds

$h = 1.9 + 0.3\cos(30 \times 8) \rightarrow 1.75$ metres

c) put $h = 2.05$ in equation

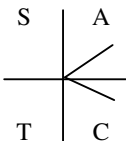
$2.05 = 1.9 + 0.3\cos(30t) \rightarrow 0.15 = 0.3\cos 30t$

$\rightarrow \cos 30t = 0.5 \rightarrow 30t = \cos^{-1} 0.5$

acute $30t = 60^\circ, 300, \dots$

so, $30t = 60$

t = 2 seconds – first time



13. a) Put $t = 10$ for October into formula.

$V = 1 + 0.5\cos(30 \times 10) \rightarrow 1.25$ million gallons

b) t can only take whole number values of 1 to 12

$V(1) = 1 + 0.5\cos(30 \times 1) \rightarrow 1.43$

$V(2) = 1 + 0.5\cos(30 \times 2) \rightarrow 1.25$

$V(3) = 1 + 0.5\cos(30 \times 3) \rightarrow 1$

$V(4) = 1 + 0.5\cos(30 \times 4) \rightarrow 0.75$

$V(5) = 1 + 0.5\cos(30 \times 5) \rightarrow 0.567$

$V(6) = 1 + 0.5\cos(30 \times 6) \rightarrow 0.5$

$V(7) = 1 + 0.5\cos(30 \times 7) \rightarrow 0.567$

$V(8) = 1 + 0.5\cos(30 \times 8) \rightarrow 0.75$

$V(9) = 1 + 0.5\cos(30 \times 9) \rightarrow 1$

$V(10)$ OK see part (a)

$V(11) = 1 + 0.5\cos(30 \times 11) \rightarrow 1.433$

$V(12) = 1 + 0.5\cos(30 \times 12) \rightarrow 1.5$

Council will need to consider water rationing in June.

See next column for an alternative solution:

Alternative solution: (Fuller understanding required)

For $t = 1, 2, 3$ and $10, 11, 12$ the cosine is positive (in 1st and 4th quadrants)

For $t = 4, 5, 6$ and $7, 8, 9$ the cosine is negative.

the minimum value will occur when $\cos = -1$ i.e $\cos 180^\circ$, then $t = 6$.

Hence $V = 1 - 0.5 = 0.5$

So, rationing needs to be considered in June

Now look at $t = 5$ and $t = 7$,

Work out V for these and you find

For May ($t = 5$) $V = 0.567$

For July ($t = 7$) $V = 0.567$ (symmetrical)

These are over critical level of 0.55 million gallons

So rationing only needs to be considered in June.