

Solutions

5 Similar Shapes

1. Linear scale factor = $\frac{9}{6} \rightarrow \frac{3}{2}$

$$\text{Volume} = 30 \times \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2} = 101.25 \text{ mls}$$

2. Linear scale factor = $\frac{40}{50} \rightarrow \frac{4}{5}$

$$\text{Area} = 3.27 \times \frac{4}{5} \times \frac{4}{5} = 2.0928 \text{ m}^2 = 2.09 \text{ m}^2$$

3. Linear scale factor = $\frac{27}{18} \rightarrow \frac{3}{2}$

$$\text{Cost} = 80 \times \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2} = \text{£} 2.70$$

4. Linear scale factor = $\frac{30}{20} \rightarrow \frac{3}{2}$

$$\text{Volume} = 0.8 \times \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2} = 2.7 \text{ litres}$$

5. Linear scale factor = $\frac{24}{30} \rightarrow \frac{4}{5}$

$$\text{Volume} = 1.2 \times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} = 0.6144 \text{ litres}$$

$$\text{Volume} = 0.61 \text{ litres (2 sig figs)}$$

6. Linear scale factor = $\frac{200}{160} \rightarrow \frac{5}{4}$

$$\text{Cost} = \text{£} 1.12 \times \frac{5}{4} \times \frac{5}{4} \times \frac{5}{4} = \text{£} 2.1875$$

$$\text{Cost} = \text{£} 2.19$$

Similar Triangles

1. Due to parallel line, Triangles are similar

$$\frac{BP}{6} = \frac{1}{1.5} \rightarrow BP = \frac{6}{1.5} = 4$$

$$\text{Hence } AP = 6 - 4 = 2 \text{ metres}$$

In Figure 2 the triangles are similar:

Let B be h metres above the ground

$$\frac{h}{1} = \frac{6}{AP} \text{ but } AP = 2 \Rightarrow \frac{h}{1} = \frac{6}{2} \Rightarrow h = 3 \text{ metres}$$

2. a) Use converse of Pythagoras in ΔABX
 $AB^2 = 300^2 = 90000$
 $AX^2 + BX^2 = 180^2 + 240^2 = 90000$
Since $AB^2 = AX^2 + BX^2$ then $\angle AXB$ is 90°
So roads AX and BX are at right angles to one another

b) Shortest route is AX \rightarrow XC \rightarrow CD
Triangles ABX and XCD are similar
 $\angle A = \angle D$, $\angle B = \angle C$ (alternate angles)

So,

$$\frac{XC}{240} = \frac{750}{300} \rightarrow XC = \frac{750 \times 240}{300} = 600$$

$$\text{Shortest distance} = 180 + 600 + 750 \text{ m} \\ = 1530 \text{ metres} = 1.53 \text{ km}$$

3. AC = 24 cm (diameter) and $\angle ACD = 58^\circ$
Using SOH-CAH-TOA,

$$\sin 58 = \frac{AD}{24} \rightarrow AD = 24 \sin 58 = 20.35 \text{ cm}$$

ΔAEO and ΔADC are similar (parallel line)

$$\frac{AE}{20.35} = \frac{12}{24} \rightarrow AE = \frac{1}{2} \times 20.35 = 10.175$$

$$\text{Hence } ED = 20.35 - 10.175 = 10.175 \text{ cm}$$

4. $\angle B = 80^\circ$ (angle sum triangle ABC)
 $\angle E = 65^\circ$ (angle sum triangle DEF)
Triangles are equiangular, hence similar.

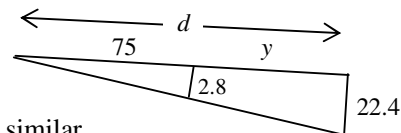
$$\frac{DE}{4.6} = \frac{10.5}{4.2} \rightarrow DE = \frac{10.5 \times 4.6}{4.2} = 11.5$$

$$\text{Hence } DE = 11.5 \text{ centimetres}$$

5. $\frac{BE}{6} = \frac{10}{12} \rightarrow BE = \frac{10 \times 6}{12} = 5 \text{ cms}$

6. $\frac{CD}{8.4} = \frac{3}{4.5} \rightarrow CD = \frac{3 \times 8.4}{4.5} = 5.6 \text{ m}$

7.



The triangles are similar (parallel line).

$$\frac{d}{75} = \frac{22.4}{2.8} \rightarrow d = \frac{22.4 \times 75}{2.8} = 600 \text{ cms}$$

$$\text{Hence } y = 600 - 75 = 525 \text{ centimetres}$$

So, distance from top of 10p coin to top of person's head is 525 centimetres.