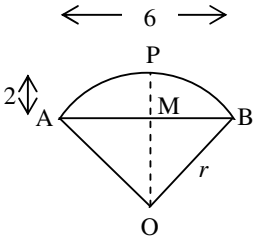
Solutions

6 Pythagoras

1. Let mid-point of AB be M and radius OB be r



MB = 3m (½ of width)
OP = r (radius)
OM = r - 2

By Pythagoras: $r^2 = (r-2)^2 + 3^2$

So, $r^2 = r^2 - 4r + 4 + 9$

4r = 13, r = 3.25 metres.

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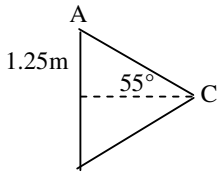
2. a) Draw triangle as shown



$\sin 55° = \frac{1.25}{AC}$

$AC = \frac{1.25}{\sin 55°} = 1.5259...$ hence AC = 1.53m (3 sf)

b) Find perimeter of table
radius of arc = 1.53m (diameter = 3.06m)
length of curved end = $\frac{110}{360} \times \pi \times 3.06 = 2.94$ m
length of straight section = 5m
Perimeter = 2.94 + 2.94 + 5 + 5 = 15.88m
= 1588 cm
No. of people = 1588 ÷ 75 = 21.17.. So **21 people**

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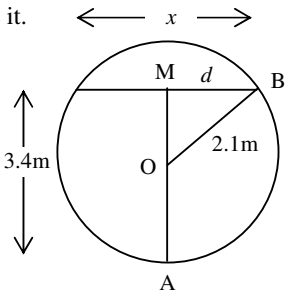
3. a) Find MB and then double it.



OA = 2.1m (radius)

OM = 3.4 – 2.1 m
= 1.3 m

By Pythagoras

$2.1^2 = 1.3^2 + d^2$

Hence $2.1^2 - 1.3^2 = d^2$ so d = 1.649..
Hence x = d × 2 = 3.298… = 3.3 m (2 s.f.)

b) By symmetry, the oil width will be the same, when it is below the centre by a distance OM.

i.e. 2.1 – 1.3 = 0.8 metres.

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4. a) CQ = radius = 10 cm
By Pythagoras: $10^2 = x^2 + 8^2$ hence $10^2 - 8^2 = x^2$
and so, x = 6 cm

b) Height of figure = Ht. of Triangle + x + 10
Let height of triangle be h, using symmetry:
$h^2 + 8^2 = 17^2$
So h = 15 cm
Height of figure = 15 + 6 + 10 cm = **31 cms**

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5. a) Use converse of Pythagroas
AB^2 = 90 000
$AX^2 + BX^2 = 180^2 + 240^2$ = 90 000
Since $AB^2 = AX^2 + BX^2$ then AXB is 90°
(converse of Pythagoras)
So AX and BX are at right angles to each other.

b) Shortest route is AX, XC, CD
Need to find XC
Triangles ABX and CXD are similar
(using alternate angles on the parallel lines AB, CD)

$\frac{XC}{240} = \frac{750}{300}$ hence XC = 600m

Length of shortest route
= 180 + 600 + 750 = 1530 metres.

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6. Use converse of Pythagoras
$d^2 = 37.3^2$ = 1391.29
$22.5^2 + 30^2 = 506.25 + 900 = 1406.25$
Since $d^2 \neq 22.5^2 + 30^2$ then AXB is NOT 90°
(converse of Pythagoras)
So No, the frame is NOT rectangular.

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7. Find length of BD: OD = radius = 60 cm
BOD is a right angled triangle
Use Pythagoras: $BD^2 = 60^2 + 60^2$
BD = 84.852…. cm = 84.9 cm (1 dp)

Perimeter of circular part of table = $\frac{270}{360} \times \pi \times 120$
= 282.7433.. cm = 282.7 cm (1 dp)

Perimeter of table = 84.9 + 282.7 = 367.6 cm

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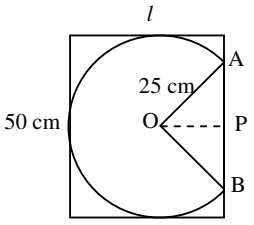
8. Area of sector = $\frac{280}{360} \times \pi \times 50$ = 122.17.. = 122.2 cm^2

To find l, we need to find OP and then add to the radius of 25 cm



Angle AOB = 360 – 280 = 80°
Angle AOP = 40°
Using SOH-CAH-TOA

$\cos 40 = \frac{OP}{25}$ hence OP = 19.151.. = 19.2 cm

so min. length l reqd. = 19.2 + 25 = **44.2 cms.**

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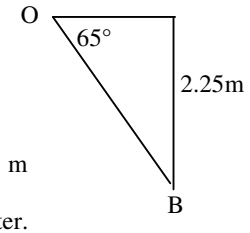
9. If they are to meet then this will be the angle in a semi-circle which should be 90°
Use converse of Pythagoras
4.1^2 = 16.81
$2.6^2 + 3.1^2 = 16.37$
Since $4.1^2 \neq 2.6^2 + 3.1^2$ then angle at top of bridge is NOT 90° (converse of Pythagoras).
So beams will NOT fit this archway.

Solutions

6 Pythagoras *(continued)*

10. a) Use SOH-CAH-TOA



$$\sin 65 = \frac{2.25}{OB}$$

hence OB = 2.482.. = 2.48 m

b) Length of border = perimeter.

radius of curved edge = 2 x 2.48 = 4.96 m

Curved length = $\frac{130}{360} \times \pi \times 4.96$ = 5.626.. m

Length of border = 8.3 + 8.3 + 4.5 + 5.6 = 26.7 m

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11. Height of tunnel
= distance O to the floor + O to top (the radius).
Let distance O to floor be d

Use Pythagoras and symmetry:

$2.5^2 = d^2 + 1.2^2$ hence $d = 2.193\ldots$

Hence height of tunnel = 2.19 + 2.5 = 4.7 m (2 sf)

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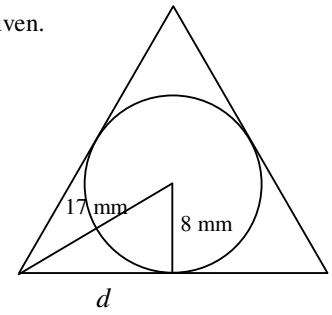
6. First draw the diagram
Mark on information given.



Find d.

$17^2 = d^2 + 8^2$

d = 15 cm

Hence each side
of triangle = 30 cm

Since equilateral, perimeter = 3 x 30 = 90 cms.

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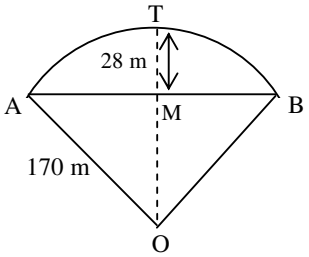
10. Converse of Pythagoras
14.5^2 = 210.25
$11.6^2 + 8.7^2 = 210.25$
Since $14.5^2 = 11.6^2 + 8.7^2$ then
angle is a perfect right angle (converse of Pythagoras),
so yes, it will be acceptable.

11.



OT = 170 (radius)
OM = 170 – 28 = 142

Use Pythagoras

$170^2 = AM^2 + 142^2$

AM = 93.466….

Hence AB
= 2 x 93.466..
= 187 m (3 sf)

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12. Converse of Pythagoras
AC = 6 – (2 + 2.5) = 1.5 m
AB^2 = 2.5^2 = 6.25
$AC^2 + CB^2 = 2^2 + 1.5^2 = 6.25$
Since $AB^2 = AC^2 + CB^2$ then angle ACB = 90°
(converse of Pythagoras).

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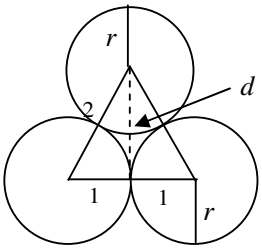
13. Height of stacked pipes =
radius + radius
+ distance between centres
of two layers



Triangle is equilateral,
by symmetry
and sides are all 2 m

By Pythagoras: $2^2 = 1^2 + d^2$ so d = 1.732… m

Hence height of pipe stack = 1 + 1 + 1.73 **= 3.73 m**

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12. a) If AB = 2 then BC = 2 (it is a square)
By Pythagoras: $AC^2 = 2^2 + 2^2$ so AC = $\sqrt{8} \to 2\sqrt{2}$

b) In any square of side a.

Diagonal = $\sqrt{a^2 + a^2} = \sqrt{2a^2} = a\sqrt{2}$

Ratio of side to diagonal is: a : a√2
which is 1 : √2

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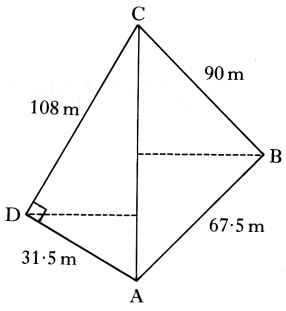
13. Since angle ADC = 90°
By Pythagoras
$AC^2 = 108^2 + 31.5^2$
= 12656.25
$AB^2 + BC^2 = 90^2 + 67.5^2$
= 12656.25
Hence angle ABC = 90°
(converse of Pythagoras)



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14. a) If d = 2, then using Pythagoras

$R^2 = 6^2 + (R-2)^2$ $\quad R^2 = 36 + R^2 - 4R + 4$

Hence 4R = 40 and so R = 10

b) Volume = Volume of Cap + cylinder

Cap: $V = \frac{1}{3}\pi(2)^2(3\times 10 - 2) = \frac{112}{3}\pi$

Cylinder: $V = \pi \times \frac{5}{2} \times \frac{5}{2} \times \frac{8}{1} = 50\pi$

Total volume = $\frac{150}{3}\pi + \frac{112}{3}\pi = \frac{262\pi}{3}$
