

X100/301

NATIONAL
QUALIFICATIONS
2006

FRIDAY, 19 MAY
9.00 AM – 10.10 AM

MATHEMATICS
HIGHER

Units 1, 2 and 3

Paper 1

(Non-calculator)

Read Carefully

- 1 Calculators may **NOT** be used in this paper.
- 2 Full credit will be given only where the solution contains appropriate working.
- 3 Answers obtained by readings from scale drawings will not receive any credit.



FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Scalar Product: $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$, where θ is the angle between \mathbf{a} and \mathbf{b}

or $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

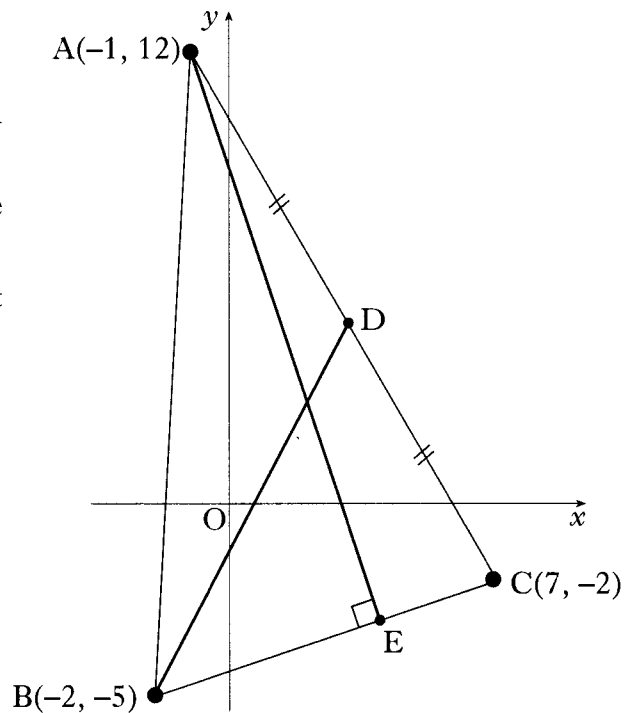
$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

ALL questions should be attempted.

Marks

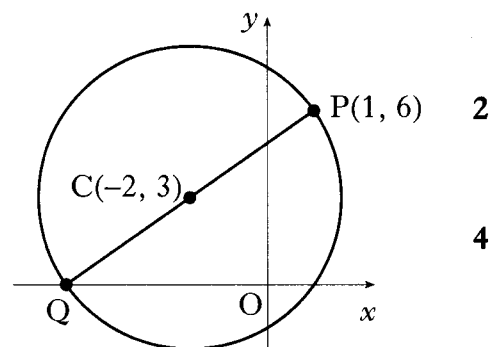
1. Triangle ABC has vertices A(-1, 12), B(-2, -5) and C(7, -2).

- (a) Find the equation of the median BD.
- (b) Find the equation of the altitude AE.
- (c) Find the coordinates of the point of intersection of BD and AE.



2. A circle has centre C(-2, 3) and passes through P(1, 6).

- (a) Find the equation of the circle.
- (b) PQ is a diameter of the circle. Find the equation of the tangent to this circle at Q.



3. Two functions f and g are defined by $f(x) = 2x + 3$ and $g(x) = 2x - 3$, where x is a real number.

- (a) Find expressions for:

(i) $f(g(x))$;

(ii) $g(f(x))$.

- (b) Determine the least possible value of the product $f(g(x)) \times g(f(x))$.

3

2

[Turn over

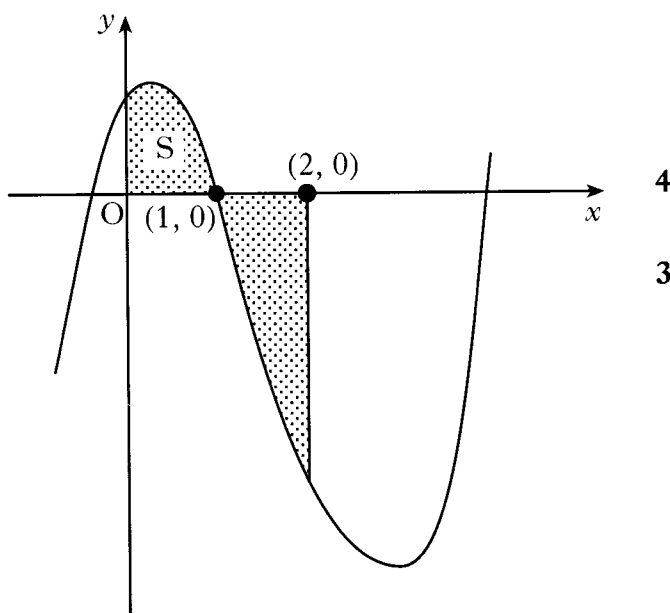
4. A sequence is defined by the recurrence relation $u_{n+1} = 0.8u_n + 12$, $u_0 = 4$.
- (a) State why this sequence has a limit. 1
- (b) Find this limit. 2

5. A function f is defined by $f(x) = (2x - 1)^5$.
- Find the coordinates of the stationary point on the graph with equation $y = f(x)$ and determine its nature. 7

6. The graph shown has equation $y = x^3 - 6x^2 + 4x + 1$.

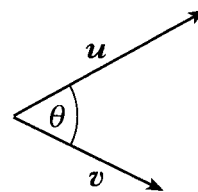
The total shaded area is bounded by the curve, the x -axis, the y -axis and the line $x = 2$.

- (a) Calculate the shaded area labelled S.
- (b) Hence find the total shaded area.

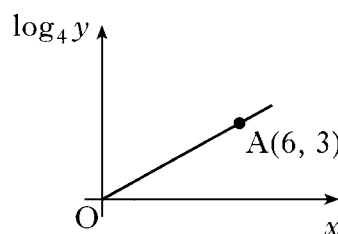


7. Solve the equation $\sin x^\circ - \sin 2x^\circ = 0$ in the interval $0 \leq x \leq 360$. 4
8. (a) Express $2x^2 + 4x - 3$ in the form $a(x + b)^2 + c$. 3
- (b) Write down the coordinates of the turning point on the parabola with equation $y = 2x^2 + 4x - 3$. 1

9. \mathbf{u} and \mathbf{v} are vectors given by $\mathbf{u} = \begin{pmatrix} k^3 \\ 1 \\ k+2 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 1 \\ 3k^2 \\ -1 \end{pmatrix}$, where $k > 0$.



- (a) If $\mathbf{u} \cdot \mathbf{v} = 1$, show that $k^3 + 3k^2 - k - 3 = 0$. 2
- (b) Show that $(k + 3)$ is a factor of $k^3 + 3k^2 - k - 3$ and hence factorise $k^3 + 3k^2 - k - 3$ fully. 5
- (c) Deduce the only possible value of k . 1
- (d) The angle between \mathbf{u} and \mathbf{v} is θ . Find the exact value of $\cos \theta$. 3
10. Two variables, x and y , are connected by the law $y = a^x$. The graph of $\log_4 y$ against x is a straight line passing through the origin and the point A(6, 3). Find the value of a . 4



[END OF QUESTION PAPER]