Rules of differentiation

Sum rule: if u and v are functions of x, then

$$\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}.$$

Constant multiplier rule: if k is a constant and u is a function of x, then

$$\frac{d}{dx}(ku) = k\frac{du}{dx}.$$

Product and quotient rules: if u and v are functions of x, then

$$\frac{d}{dx}(uv) = \frac{du}{dx}v + u\frac{dv}{dx},$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \left(\frac{du}{dx}v - u\frac{dv}{dx}\right)\Big/v^2.$$

Chain rule or **'function of a function' rule:** if f and g are two functions, then the derivative of their composition is given by the **function of a function rule**:

$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x).$$

A more common way to write this is the **chain rule**:

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx},$$

where (in this case) u stands for $g(x)$ and y for $f(u)$.

Standard derivatives

In each of the following cases the domain is the largest set of real numbers x for which the function is defined.

Function	Derivative
x^α	$\alpha x^{\alpha-1}$ (α any number)
$\log_e x$	$1/x$
$\log_e(-x)$	$1/x$
e^x	e^x
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

The following derivatives are given in a form that is useful for integration. The constant a is assumed positive, and in the derivatives of arcsec and arccosec it is assumed that $x > a$ (rather than $x < -a$).

Function	Derivative
$\arcsin\left(\frac{x}{a}\right)$	$\frac{1}{\sqrt{a^2-x^2}}$
$\arccos\left(\frac{x}{a}\right)$	$\frac{-1}{\sqrt{a^2-x^2}}$
$\arctan\left(\frac{x}{a}\right)$	$\frac{a}{a^2+x^2}$
$\operatorname{arccot}\left(\frac{x}{a}\right)$	$\frac{-a}{a^2+x^2}$
$\operatorname{arcsec}\left(\frac{x}{a}\right)$	$\frac{a}{x\sqrt{x^2-a^2}}$
$\operatorname{arccosec}\left(\frac{x}{a}\right)$	$\frac{-a}{x\sqrt{x^2-a^2}}$

Rules of integration

Examples

1. The fundamental theorem of calculus

$$\int f(x)\,dx = F(x) + C$$

if and only if $f(x) = \dfrac{dF(x)}{dx}$.

$$\int x\,dx = \tfrac{1}{2}x^2 + C,$$
$$x = \frac{d}{dx}\left(\tfrac{1}{2}x^2\right).$$

2. Linearity rule

$$\int [\alpha f(x) + \beta g(x)]\,dx$$
$$= \alpha \int f(x)\,dx + \beta \int g(x)\,dx.$$

$$\int \left(\tfrac{1}{2}e^x - \tfrac{1}{2}e^{-x}\right) dx$$
$$= \tfrac{1}{2}\int e^x\,dx - \tfrac{1}{2}\int e^{-x}\,dx$$
$$= \tfrac{1}{2}e^x + \tfrac{1}{2}e^{-x} + C.$$

3. Substitution rules

For integrals of the form

$$I = \int g(f(x))\frac{df(x)}{dx}\,dx,$$

substitute $u = f(x)$ and use $\dfrac{du}{dx}\,dx = du$ to obtain

$$I = \int g(u)\,du.$$

To find $\int \sin(ax+b)\,dx = I_1$, let $u = ax + b$; then $a\,dx = du$. So

$$I_1 = \frac{1}{a}\int \sin(ax+b)\,a\,dx$$
$$= \frac{1}{a}\int \sin u\,du = -\frac{1}{a}\cos u + C$$
$$= -\frac{1}{a}\cos(ax+b) + C.$$

Otherwise, find a suitable relation between u and x and use

$$dx = \frac{dx}{du}\,du.$$

To find $\displaystyle\int \frac{dx}{(1+x^2)^{3/2}} = I_2$, let $x = \tan u$; then $dx = \sec^2 u\,du$. So

$$I_2 = \int \frac{\sec^2 u\,du}{(1+\tan^2 u)^{3/2}}$$
$$= \int \frac{\sec^2 u\,du}{\sec^3 u}$$
$$= \int \cos u\,du = \sin u + C$$
$$= \sin(\arctan x) + C.$$

4. Integration by parts

Let $u = u(x)$ and $v = v(x)$; then

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx.$$

$\int f g' = fg - \int f' g$

To find $\int x\cos x\,dx = I_3$, let $u = x$ and $v = \sin x$; then

$$I_3 = \int x\frac{d}{dx}(\sin x)\,dx$$
$$= x\sin x - \int \sin x \times 1\,dx$$
$$= x\sin x + \cos x + C.$$