

PRESTWICK ACADEMY (2)

National 5 Mathematics

Applications

Unit Test Preparation Booklet

FORMULAE LIST

The roots of $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$ or $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

Area of a triangle: $A = \frac{1}{2} ab \sin C$

Volume of a sphere: $V = \frac{4}{3} \pi r^3$

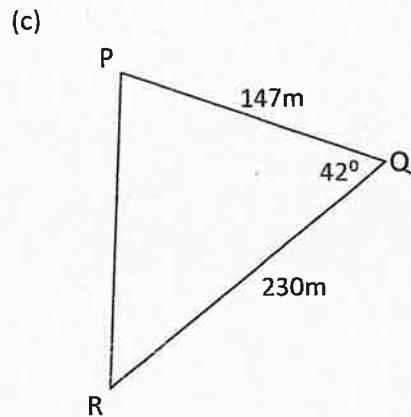
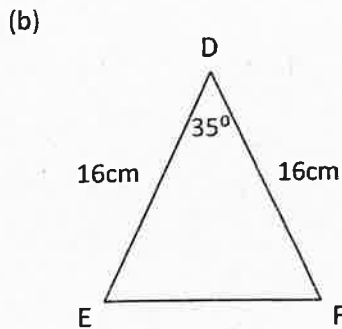
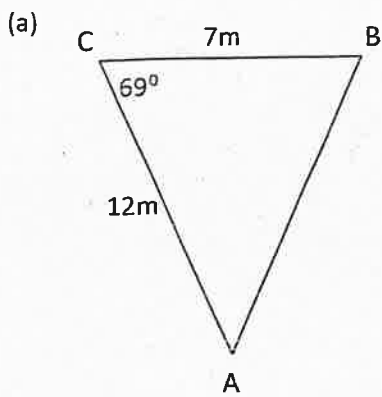
Volume of a cone: $V = \frac{1}{3} \pi r^2 h$

Volume of a pyramid: $V = \frac{1}{3} Ah$

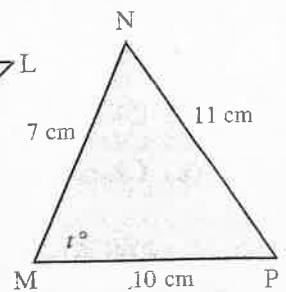
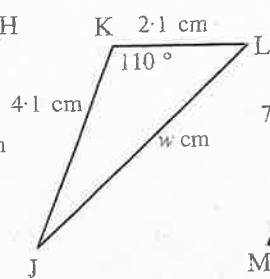
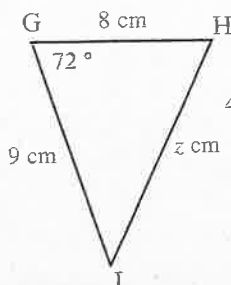
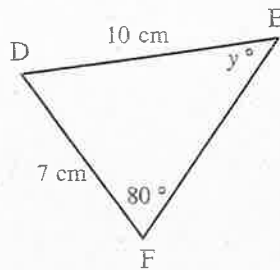
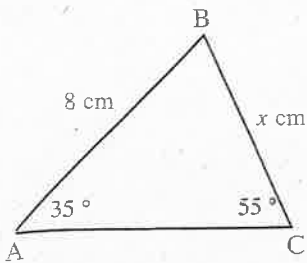
Standard deviation: $s = \sqrt{\frac{\sum(x - \bar{x})^2}{n-1}} = \sqrt{\frac{\sum x^2 - (\sum x)^2/n}{n-1}}$, where n is the sample size.

Assessment Standard 1.1

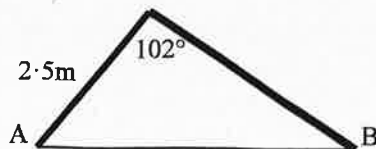
1. Calculate the area of each triangle below.



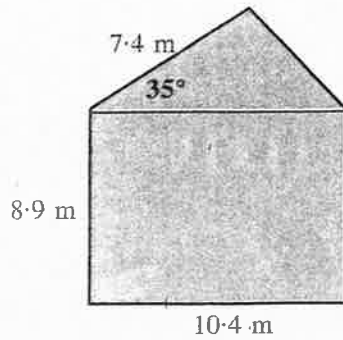
2. For each of these use the sine rule or cosine rule to calculate the values of x , y , z , w , and t .



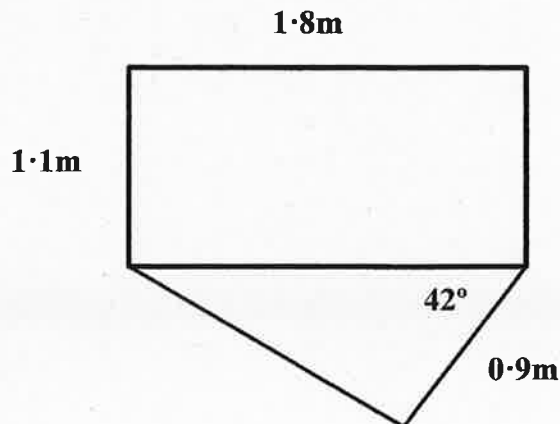
3. A pole, 6 metres long, is blown over and broken in a storm.
 The distance from the base of the pole to the 'break' is 2.5 metres.
 The angle at the 'break' is 102° .
 Calculate the distance between the top and the bottom of the pole (AB).



4. The side wall of a house, with measurements as shown, requires painting. The wall consists of a rectangular base with a triangle on top. On average, a litre of paint will cover 9 square metres. A painter estimates that he will require 12 litres of paint. Will this be enough? Justify your answer.



5. The dimensions of Karen's desk top are shown in the diagram below.



She wants to varnish the desk top.

On average, 100ml of varnish will cover 2 square metres.

She estimates that she will need 150ml of varnish.

Will this be enough?

Justify your answer.

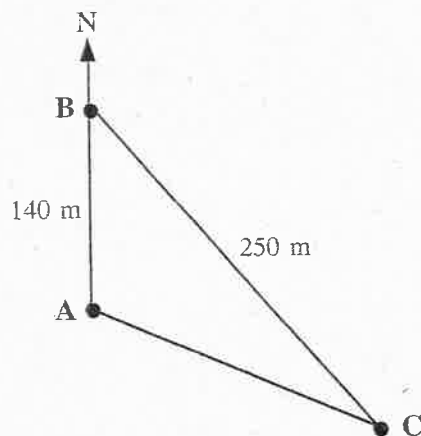
6. Three CCTV cameras, A, B and C are situated in Ayr town centre.

A is 140 metres due South of B.

B is 250 metres from C.

C is on a bearing of 120° from A.

Calculate the bearing of C from B.



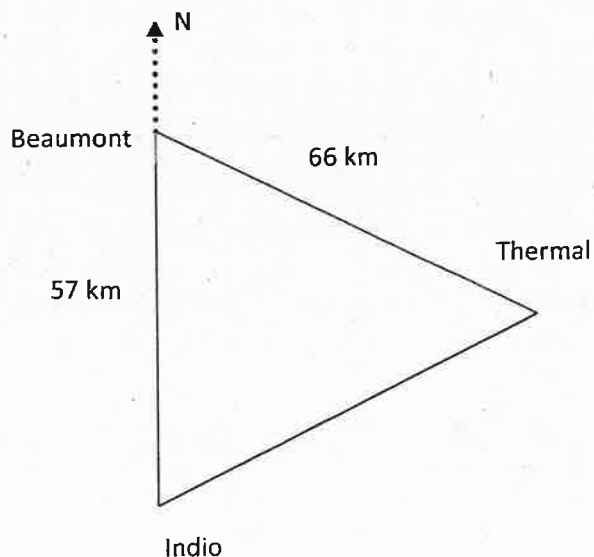
7. Three oil rigs are situated in the North Sea.

Beaumont is 57km north of Indio and is 66km from Thermal.

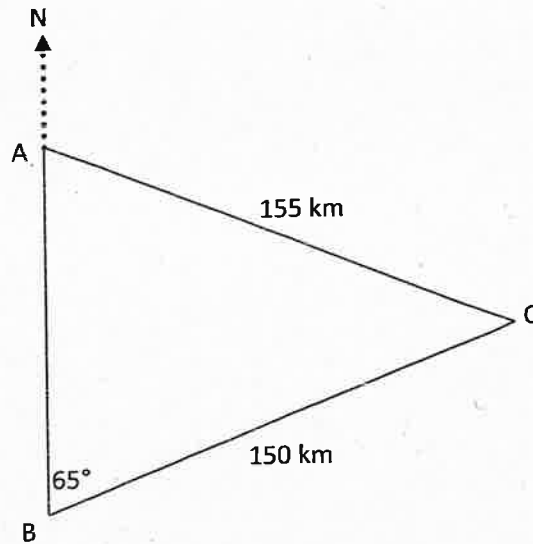
Thermal is on a bearing of 070° from Indio.

Calculate the bearing of Thermal from Beaumont.

Give your answer to the nearest degree.



8. Three ships, Arran (A), Bigga (B) and Callandish (C) are positioned as shown below.



Arran is 155km from the Callandish.

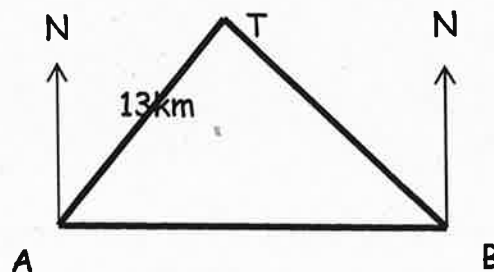
The Callandish is on a bearing of 065° from the Bigga.

The Bigga and the Callandish are also 150km from each other.

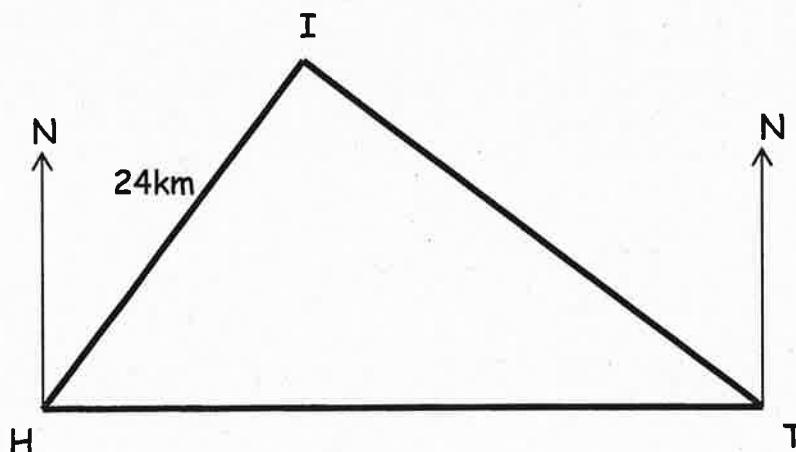
What is the bearing of the Callandish from Arran?

Give your answer to the nearest degree.

9. A coastguard at A is due West of a coast guard at B.
A tanker is spotted at T, on a bearing of 042° from A and on a bearing of 314° from B.
A is 13km from the tanker.
How far is the tanker from the coastguard at B.



10. A boat sails from a harbour H, via an island I, to a town T, as shown in the diagram. The distance from the harbour to the island is 24km.

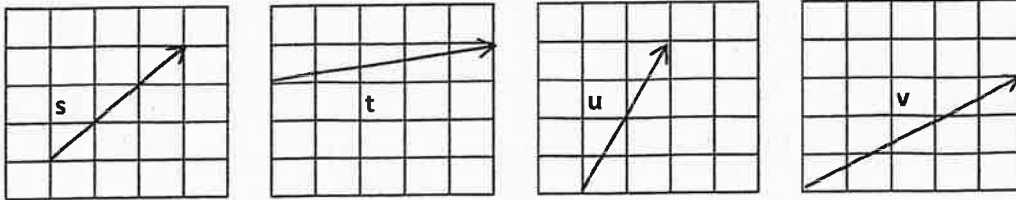


The boat sails on a bearing of 035° from the harbour to the island.
The bearing of the island from the town is 300° .
The town is due East of the harbour.

Calculate the distance from the island to the town.

Assessment Standard 1.2

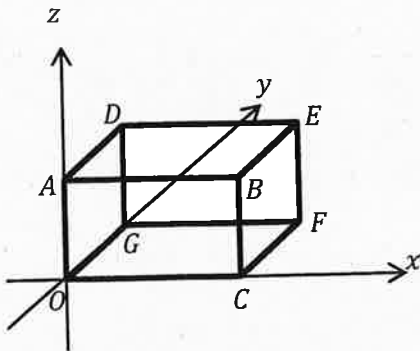
1. The diagrams below show four vectors s , t , u and v .



On squared paper, draw the resultant vector for

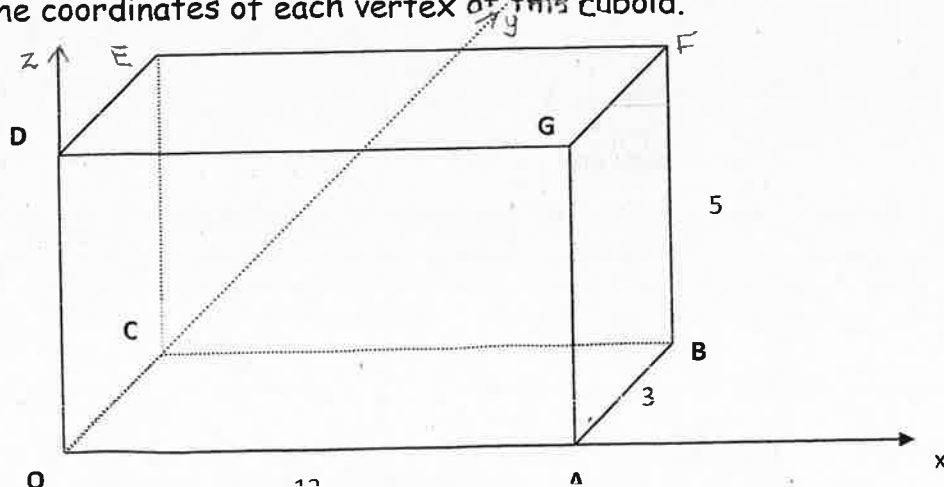
- (a) $s + t$ (b) $2t + u$ (c) $v + 3s$ (d) $4u + 2v$

2. The diagram shows a cuboid.
The coordinates of point E are (5, 3, 1).

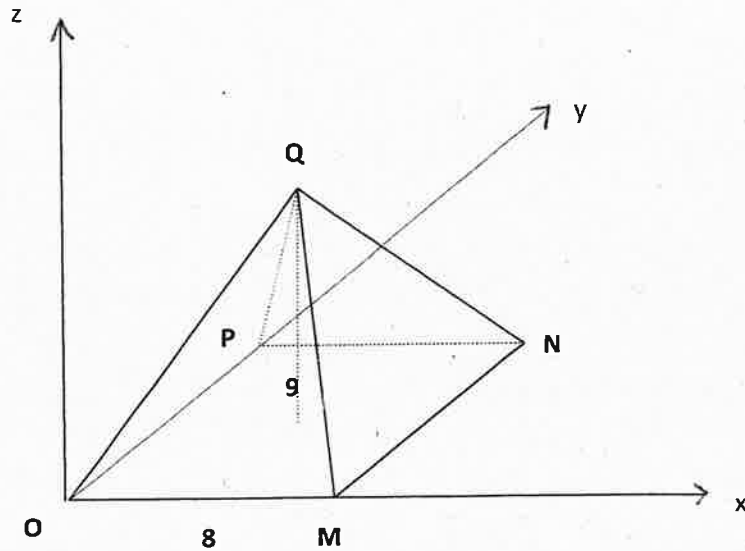


Find the coordinates of each vertex of the cuboid.

3. State the coordinates of each vertex of this cuboid.



4. State the coordinates of each vertex of this square based pyramid.



5. Vectors $a = \begin{pmatrix} 6 \\ 8 \end{pmatrix}$, $b = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$, $c = \begin{pmatrix} 9 \\ -10 \end{pmatrix}$ and $d = \begin{pmatrix} 0.5 \\ -0.5 \end{pmatrix}$.

Calculate (a) $|2a + 3b|$ (b) $|c - 12d|$ (c) $|2a - 4d|$

6. $u = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$, and $v = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$.

(a) Find the resultant vector $u + 3v$.

(b) Find $|u + 3v|$.

7. (a) The forces acting on a body are represented by three vectors u , v and w .

$$u = \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix}, \quad v = \begin{pmatrix} 11 \\ 0 \\ -4 \end{pmatrix} \quad \text{and} \quad w = \begin{pmatrix} -3.5 \\ 4.5 \\ 6 \end{pmatrix}.$$

Find the resultant force.

(b) The forces acting on a body are represented by three vectors p , q and r .

$$p = \begin{pmatrix} -0.5 \\ 7.5 \\ -2.5 \end{pmatrix}, \quad q = \begin{pmatrix} -5.5 \\ -1 \\ -9.5 \end{pmatrix} \quad \text{and} \quad r = \begin{pmatrix} 7.5 \\ -1.5 \\ 13 \end{pmatrix}.$$

Find the resultant force.

(c) The forces acting on a body are represented by three vectors a , b and c .

$$a = \begin{pmatrix} -6 \cdot 5 \\ 2 \\ -8 \end{pmatrix}, \quad b = \begin{pmatrix} 10 \\ -4 \cdot 5 \\ 8 \cdot 5 \end{pmatrix} \quad \text{and} \quad c = \begin{pmatrix} -3 \cdot 5 \\ 2 \cdot 5 \\ 3 \end{pmatrix}.$$

Find the resultant force.

8. The vector u has components $\begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}$ and vector v has components $\begin{pmatrix} 2 \\ -4 \\ 1 \end{pmatrix}$.

Calculate $|4u - 2v|$.

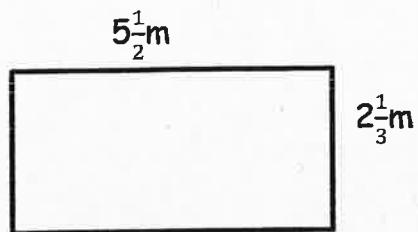
Assessment Standard 1.3

1. A house which cost £43 000 four years ago has appreciated in value by 1.5% each year since then.
Calculate the current value of the house.
2. I bought a piano for £3000 three years ago. Its value has depreciated by 18% each year since then.
Calculate the current value of the piano.
3. It is estimated that an iceberg weighs 84 000 tonnes.
As the iceberg moves into warmer water, its weight decreases by 25% each day.
What will the iceberg weigh after 3 days in the warmer water?
4. Bacteria in a test tube increase at a rate of 4.6% per hour.
At 12 noon, there are 50 000 bacteria.
How many will there be at 5pm?
5. A food processor has been reduced in price by 20% and now costs £120.
What was the original price of the processor?
6. The value of a bungalow has appreciated by 6% in the last year.
It is now valued at £180 000.
What was its value last year?
Give your answer to the nearest pound.
7. Ben measured the tree in the local park and found it to be 12 metres tall.
He told Bill that this was a 15% increase in height since last year.
Bill thinks the tree must have been 10.7 metres high last year.
Is he correct?
Explain your answer.

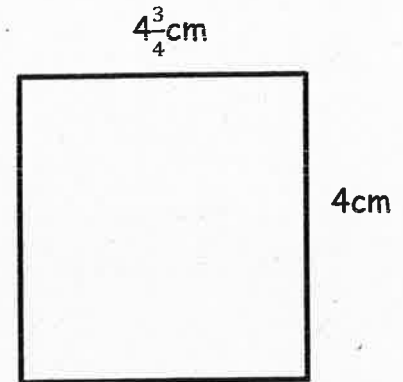
8. Evaluate
- (a) $3\frac{1}{6} \div 1\frac{2}{3}$ (b) $4\frac{1}{3} - 1\frac{1}{2}$ (c) $2\frac{2}{3} - 1\frac{4}{5}$ (d) $\frac{2}{3} \div 1\frac{1}{3}$
- (e) $2\frac{2}{5} + 3\frac{2}{3}$ (f) $1\frac{1}{2} \times 2\frac{1}{6}$ (g) $5\frac{4}{5} \times 2$ (h) $4\frac{4}{5} \div 3$

9. Calculate the area of the following shapes.

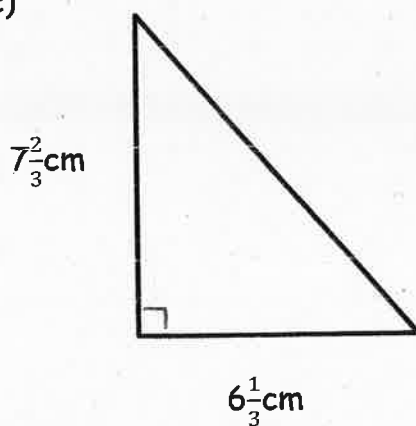
(a)



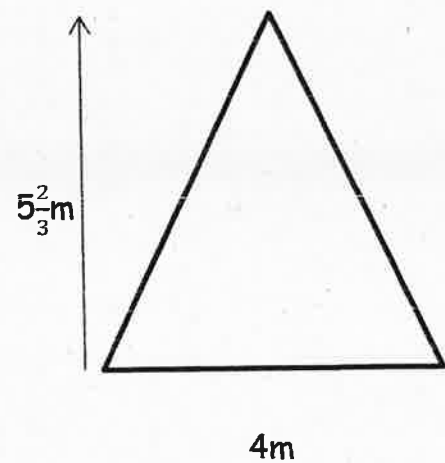
(b)



(c)



(d)



Assessment Standard 1.4

1. Two groups of six students are given the same test.

The marks for group A are...

73 47 59 71 48 62

- (a) Calculate the mean and standard deviation for group A.
- (b) In group B, the mean is 60 and the standard deviation is 29.8.
Compare the two groups and make two valid comparisons.

2. A machine is used to put drawing pins into boxes.

A sample of 8 boxes is taken and the number of drawing pins in each is counted.

The results are shown below...

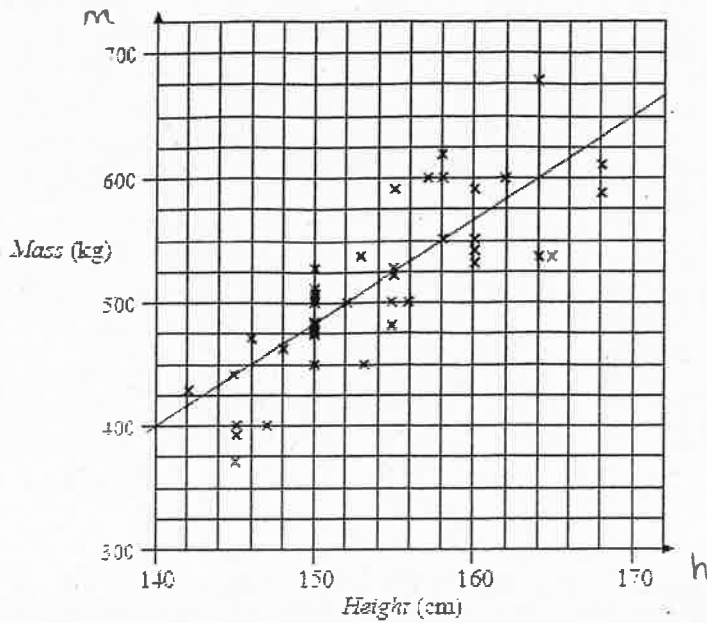
102 102 101 98 99 101 103 102

- (a) Calculate the mean and standard deviation of this sample.
- (b) A sample of 8 boxes is taken from another machine.

This sample has a mean of 103 and a standard deviation of 2.1.

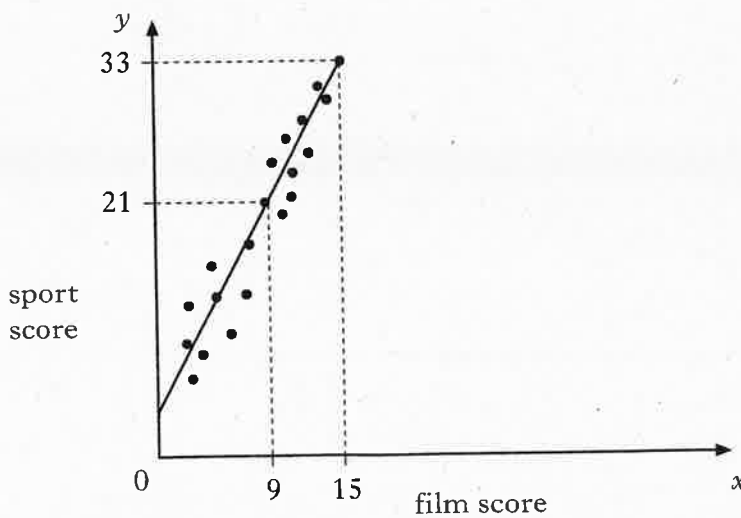
Write down two valid comparisons between the samples.

3. The scatter graph shows the heights and masses of some horses.
The scatter diagram shows the line of best fit.



- Determine the y-intercept and the gradient of the line.
- Use these values to determine the equation of the line.
- The height of a horse is 153cm, use your equation to estimate its mass.

4. Teams in a quiz answer questions on film and sport.
This scatter graph shows the scores of some of the teams.

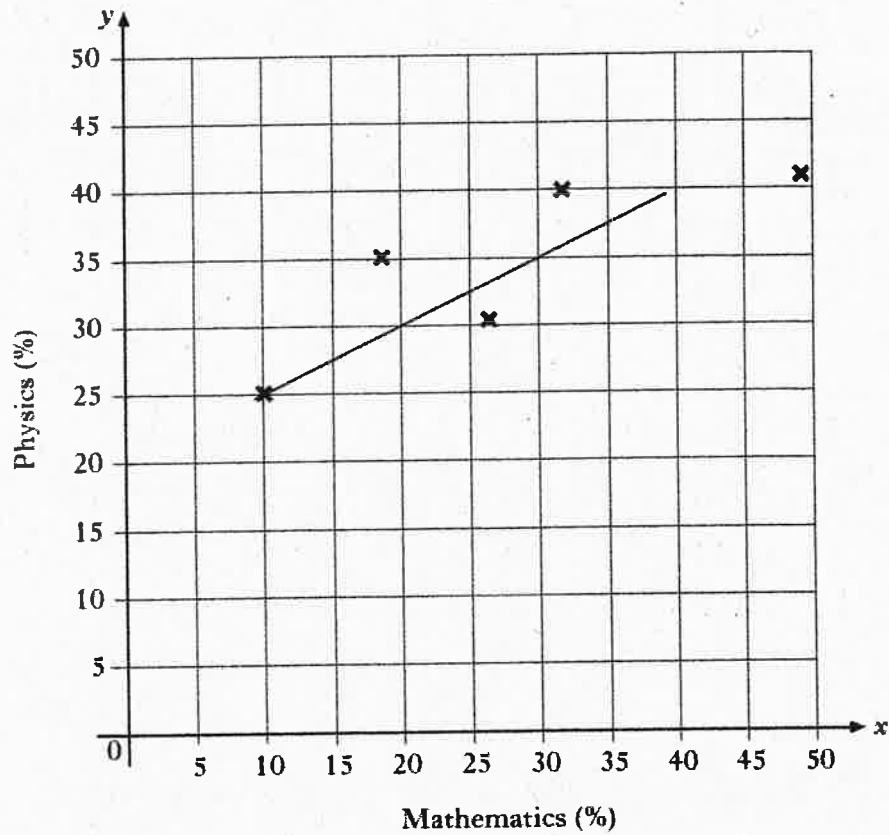


- Find the equation of the line of best fit.
- Use this to estimate the sport score for a team with film score of 20.

5.

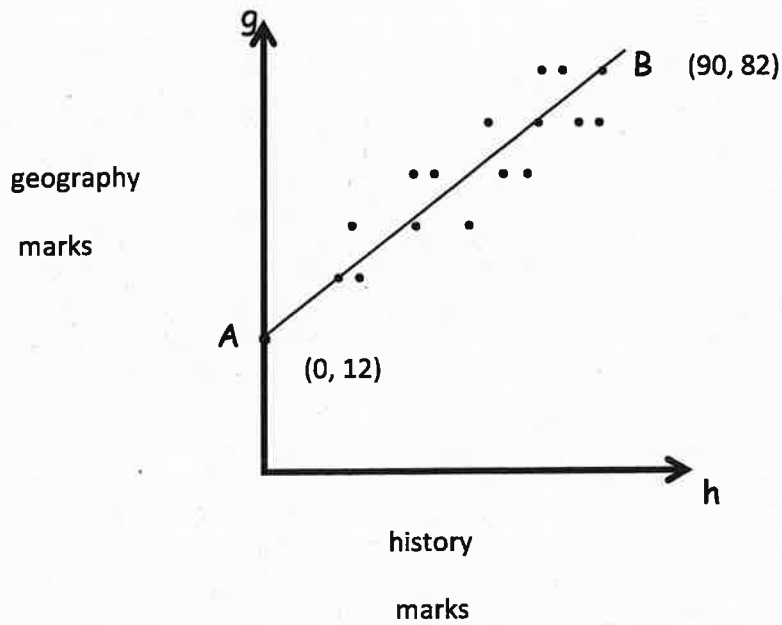
A group of students sat tests in mathematics and physics and their results are shown on the scattergraph below.

The results from the test were plotted in the scatter graph below.



- (a) Determine the gradient and the y -intercept of the line of best fit shown.
- (b) Using these values for the gradient and the y -intercept, write down the equation of the line.
- (c) Another pupil scored 64% in the mathematics test but was absent from the physics test.
Use your answer to part (b) to predict her physics mark.

6. The graph below shows the relationship between the history and geography marks of a class of students.



A best fitting straight line , AB is drawn.

Point A represents 0 marks for history and 12 marks for geography.

Point B represents 90 marks for history and 82 marks for geography.

- (a) Find the gradient and y-intercept of the line AB.
- (b) Find the equation of the straight line AB in terms of h and g.

ANSWERS

ASSESSMENT STANDARD 1.1

1. (a) 39.2 m^2 (b) 73.4 m^2 (c) 11311.7 m^2

2. (a) 5.6 cm (b) 43.6° (c) 10.0 cm (d) 5.2 cm (e) 78.5°

3. 4.7 m

4. Area of triangle = 22.1 m^2 Area of rectangle = 92.56 m^2 Total = 114.66 m^2

12 litres will cover 108 m^2 .
Area to be painted is 114.66 m^2 .
No, this is not enough.

5. Area of triangle = 0.54 m^2 Area of rectangle = 1.98 m^2 Total = 2.52 m^2

150ml will cover 3 m^2 .
Area to be varnished is 2.52 m^2 .
Yes, this is enough.

6. 149°

7. 124°

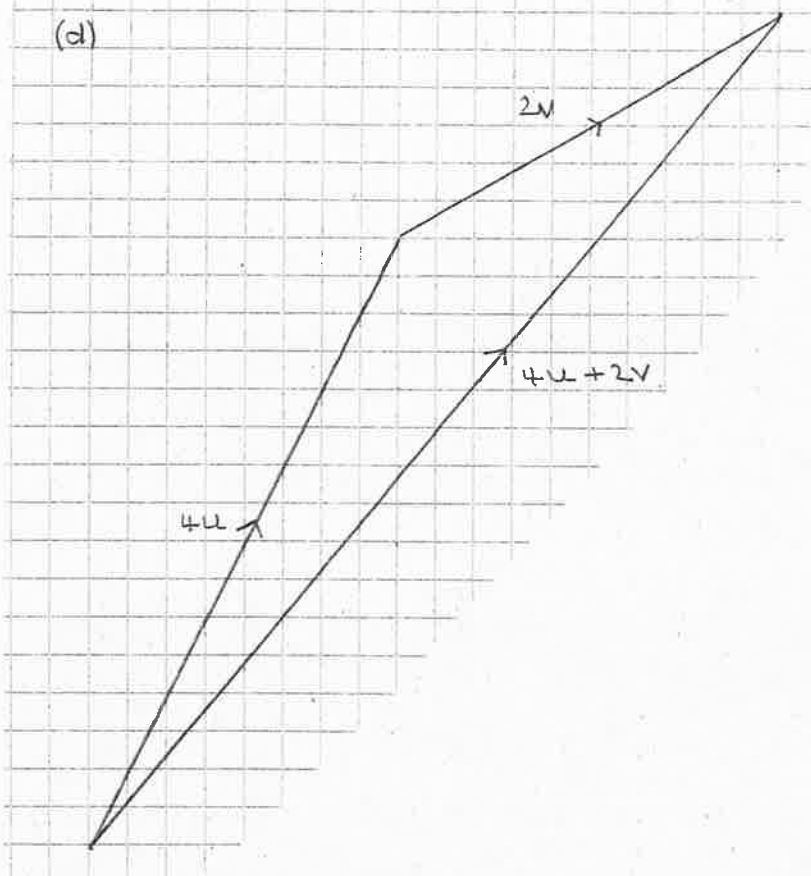
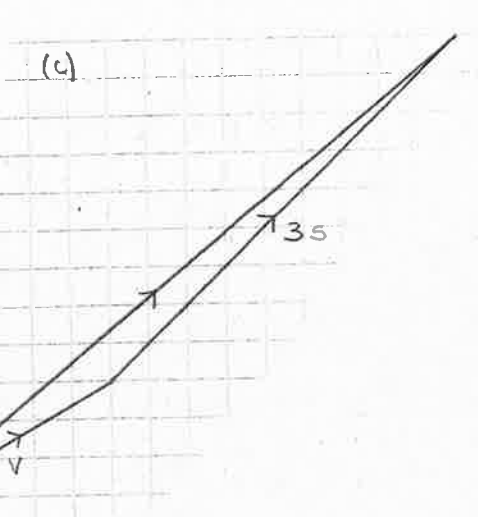
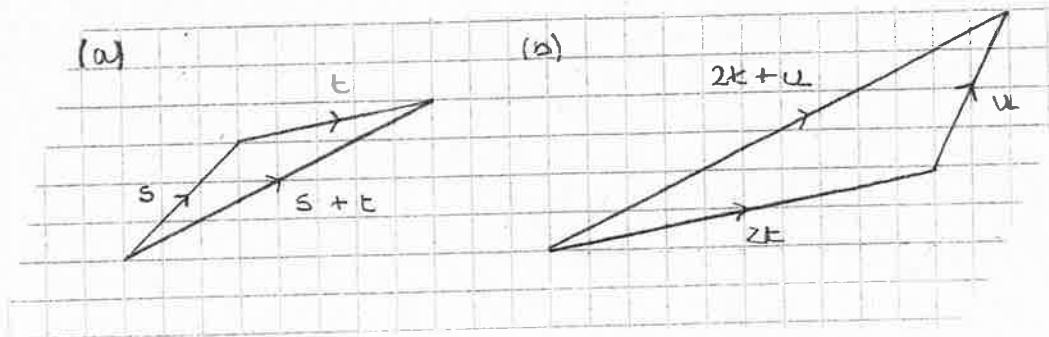
8. 119°

9. 13.91 km

10. 39.32 km

ASSESSMENT STANDARD 1.2

1.



2. $A = (0, 0, 1)$ $B = (5, 0, 1)$ $C = (5, 0, 0)$ $D = (0, 3, 1)$ $E = (5, 3, 1)$
 $F = (5, 3, 0)$ $G = (0, 3, 0)$ $O = (0, 0, 0)$

3. $A = (12, 0, 0)$ $B = (12, 3, 0)$ $C = (0, 3, 0)$ $D = (0, 0, 5)$ $E = (0, 3, 5)$
 $F = (12, 3, 5)$ $G = (12, 0, 5)$ $O = (0, 0, 0)$

4. $O = (0, 0, 0)$ $M = (8, 0, 0)$ $N = (8, 8, 0)$ $P = (0, 8, 0)$ $Q = (4, 4, 9)$

5. (a) 26 (b) 5 (c) 20·6

6. (a) $\begin{pmatrix} -10 \\ 4 \end{pmatrix}$ (b) 10·8

7. (a) $\begin{pmatrix} 12 \cdot 5 \\ 6 \cdot 5 \\ 3 \end{pmatrix}$ (b) $\begin{pmatrix} 1 \cdot 5 \\ 5 \\ 1 \end{pmatrix}$ (c) $\begin{pmatrix} 0 \\ 0 \\ 3 \cdot 5 \end{pmatrix}$

8. 10

ASSESSMENT STANDARD 1.3

1. £45 638·63

2. £1 654·10

3. 35437·5 tonnes

4. 62607·8 bacteria

5. £150

6. £169 811

7. 115% of last year = 12

.....

He is not correct. Last year the tree was 10·4 m tall.

8. (a) $\frac{19}{10}$ (b) $2\frac{5}{6}$ (c) $\frac{13}{15}$ (d) $\frac{1}{2}$

(e) $6\frac{1}{15}$ (f) $\frac{13}{4}$ (g) $\frac{58}{5}$ (h) $\frac{8}{5}$

9. (a) $\frac{77}{6} \text{ m}^2$ (b) 19 cm^2 (c) $\frac{437}{18} \text{ cm}^2$ (d) $\frac{34}{3} \text{ m}^2$

ASSESSMENT STANDARD 1.4

1. (a) mean = 60 standard deviation = 11.03

The means of the groups are equal so overall the groups did equally well in the test.

The standard deviation of group B is higher so the results for group B are more varied.

2. (a) mean = 101 standard deviation = 1.7

The mean for the second machine is higher so on average it puts a higher number of pins into the boxes.

The standard deviation for the second machine is higher so the number of pins being put into a box by the second machine is more varied than the first machine.

3. (a) Using the points (146, 450) and (164, 600) gradient = $\frac{25}{3}$ y intercept = 400

(b) $m = \frac{25}{3}h + 400$

(c) 1675 kg

4. (a) Using the points (15, 33) and (9, 21) gradient = 2
Use $y - b = m(x - a)$ to find equation of line $y = 2x + 3$

(b) 43

5. (a) Using the points (10, 25) and (30, 35) gradient = $\frac{1}{2}$
Extend the line to find y intercept = 20

(b) $y = \frac{1}{2}x + 20$

(c) 52%

6. (a) Using the points (0, 12) and (90, 82) gradient = $\frac{7}{9}$ y intercept = 12

(b) $g = \frac{7}{9}h + 12$