

**PRESTWICK ACADEMY**

**National 5 Mathematics**

**Expressions and Formulae**

**Unit Test Preparation Booklet**

## FORMULAE LIST

The roots of are  $ax^2 + bx + c = 0$   $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Sine rule:  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule:  $a^2 = b^2 + c^2 - 2bc \cos A$  or  $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

Area of a triangle:  $A = \frac{1}{2}ab \sin C$

Volume of a Sphere:  $V = \frac{4}{3}\pi r^3$

Volume of a cone:  $V = \frac{1}{3}\pi r^2 h$

Volume of a pyramid:  $V = \frac{1}{3}Ah$

Standard deviation:  $s = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}} = \sqrt{\frac{\sum x^2 - (\sum x)^2/n}{n-1}}$ , where  $n$  is the sample size.

## Assessment Standard 1.1

1. Simplify, giving your answer in surd form.

(a)  $\sqrt{12}$

(b)  $\sqrt{27}$

(c)  $\sqrt{32}$

(d)  $\sqrt{75}$

(e)  $\sqrt{20}$

(f)  $\sqrt{45}$

(g)  $\sqrt{44}$

(h)  $\sqrt{300}$

(i)  $\sqrt{125}$

(j)  $\sqrt{72}$

2. Simplify

(a)  $y^4 \times y^3$

(b)  $4m^2 \times 3m^3$

(c)  $g^6 \div g^2$

(d)  $(h^3)^2$

(e)  $\frac{x^4}{x}$

(f)  $\frac{12x^5}{4x^2}$

(g)  $\frac{h^7 \times h^{-2}}{h^3}$

(h)  $y^9 \times (y^5)^{-2}$

(i)  $2m^3 \times 4m^{-1/2}$

(j)  $(2m^3)^3$

3. (a) There are  $3.1 \times 10^7$  seconds in a solar year.  
How many seconds are there in 5 solar years?  
Give your answer in scientific notation.

(b) The Lotto jackpot of  $\text{£}8.4 \times 10^6$  was shared equally among 3 winners.  
How much did each winner receive?  
Give your answer in scientific notation.

(c) A bee weighs approximately  $1.98 \times 10^{-4}$  kilograms.  
A newly hatched baby eagle is 93 times heavier.  
Calculate the weight of the eagle.  
Give your answer in scientific notation.

## Assessment Standard 1.2

1. Expand and simplify where appropriate.

(a)  $5(x + 4)$

(b)  $4(2x - 3y)$

(c)  $a(3a + b)$

(d)  $m(5m - 6)$

(e)  $y(2y - 6)$

(f)  $y(2y + x)$

(g)  $(x + 3)(x + 5)$

(h)  $(m + 4)(m - 3)$

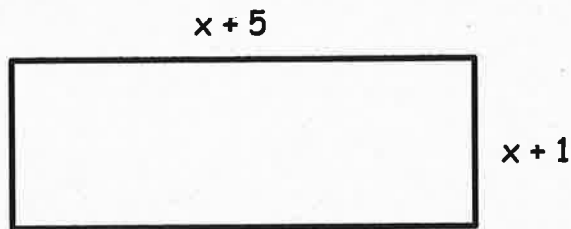
(i)  $(h - 5)(h - 3)$

(j)  $(x - 4)(x + 5)$

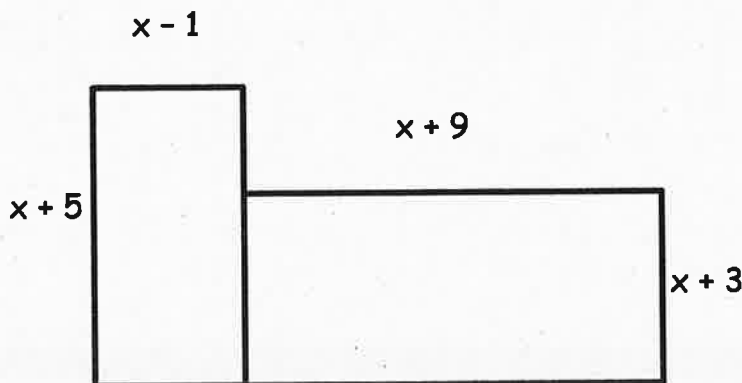
(k)  $(x + 6)^2$

(l)  $(x - 5)^2$

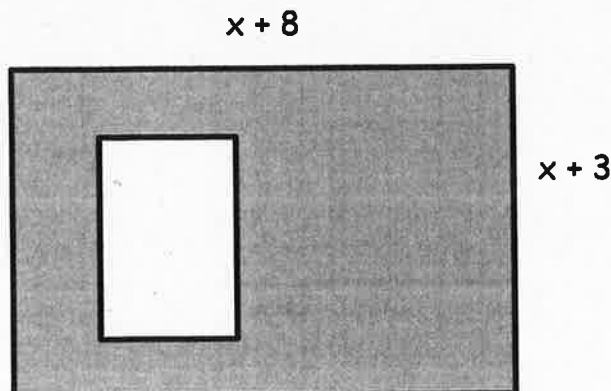
2. Find the area of this rectangle in terms of  $x$ .



3. Show that the area of this composite shape is given by  $2x^2 + 16x + 22$ .



4. The large rectangle below has a small rectangle ( $4m$  by  $3m$ ) cut out of it. Show that the shaded area of the shape below is given by  $x^2 + 11x + 12$ .



5. Factorise the following

(a)  $2x + 6$

(b)  $14y - 7$

(c)  $4a + 6b$

(d)  $10x - 25y$

(e)  $y^2 + 7y$

(f)  $a^2 - 3a$

(g)  $3d^2 + 5d$

(h)  $6m^2 - 3m$

(i)  $x^2 - 16$

(j)  $y^2 - 25$

(k)  $a^2 - 100$

(l)  $p^2 - 1$

(m)  $x^2 + 6x + 5$

(n)  $y^2 + 5y + 6$

(o)  $h^2 + 8h + 15$

(p)  $x^2 + 8x + 12$

6. Express the following in the form  $(x + p)^2 + q$

(a)  $x^2 + 6x + 4$

(b)  $x^2 + 8x + 1$

(c)  $x^2 + 8x - 10$

(d)  $x^2 + 4x + 1$

(e)  $x^2 + 10x - 8$

(f)  $x^2 + 2x - 3$

## Assessment Standard 1.3

1. Write each of these algebraic fractions in its simplest form.

$$(a) \frac{(x+4)(x+6)}{x+6}$$

$$(b) \frac{x-4}{(x-5)(x-4)}$$

$$(c) \frac{(x-6)^3}{(x-6)(x+4)}$$

$$(d) \frac{(x+6)(x-7)}{(x-7)^2}$$

$$(e) \frac{(3x+5)^2}{(3x+5)(2x+1)}$$

$$(f) \frac{(4x-7)(2x+1)}{(4x-7)^2}$$

2. Write each of the following as a single fraction.

$$(a) \frac{3}{5} + \frac{5}{8}$$

$$(b) \frac{6}{x} + \frac{3}{y}$$

$$(c) \frac{4}{a} + \frac{3}{b}$$

$$(d) \frac{8}{x} - \frac{3}{y}$$

$$(e) \frac{2}{m} - \frac{3}{n}$$

$$(f) \frac{3}{4} \times \frac{5}{7}$$

$$(g) \frac{6}{7} \times \frac{2}{3}$$

$$(h) \frac{4}{x} \times \frac{3}{y}$$

$$(i) \frac{4}{a} \times \frac{b}{c}$$

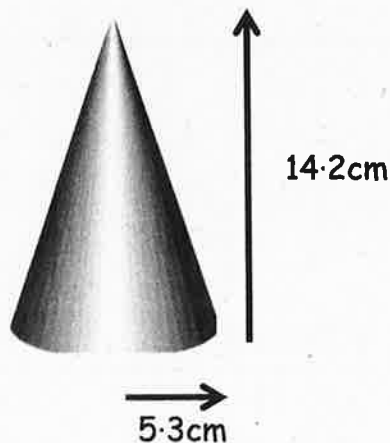
$$(j) \frac{5}{6} \div \frac{2}{3}$$

$$(k) \frac{c}{d} \div \frac{4}{b}$$

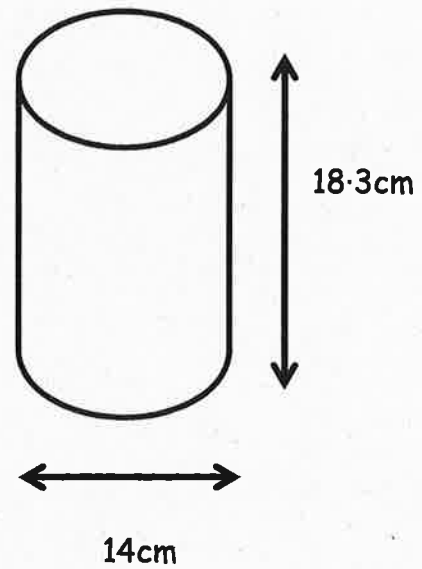
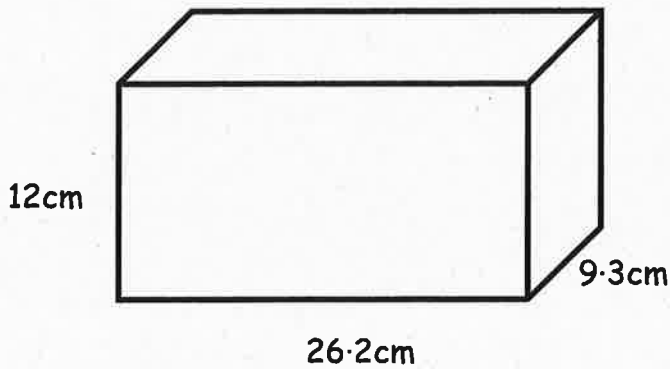
$$(l) \frac{x}{5} \div \frac{y}{z}$$

## Assessment Standard 1.4

- Points A and B have coordinates (6, 3) and (8, 4) respectively.  
Calculate the gradient of AB.
  - Points C and D have coordinates (4, 6) and (6, 10) respectively.  
Calculate the gradient of CD.
  - Which of these lines is steeper?  
Explain your answer.
- Points E and F have coordinates (-2, 1) and (1, 4) respectively.  
Calculate the gradient of EF.
  - Points G and H have coordinates (-4, 2) and (6, 4) respectively.  
Calculate the gradient of GH.
  - Which of these lines is steeper?  
Explain your answer.
- Find the gradient of the line joining K (4, -3) to L (-1, 9).
- Find the gradient of the line joining M (-4, -3) to N (-1, -7).
- Calculate the volume of the following.  
Give your answer correct to 2 significant figures.



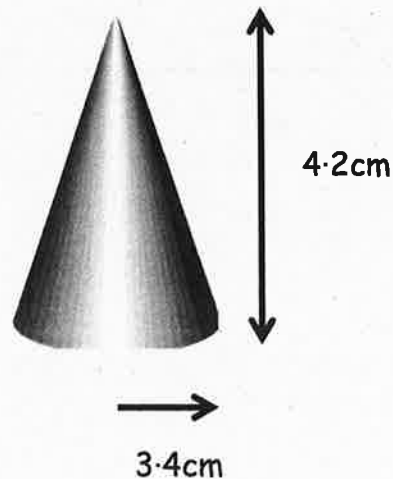
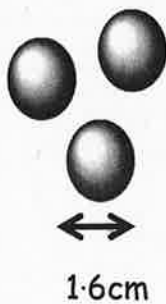
6. A tank in the shape of a cuboid is full of water.  
The water is poured into a cylinder shaped tub.  
The dimensions of the containers are shown in the diagrams below.



Will the tub hold all the water from the tank or will it overflow?

Give a reason for your answer.

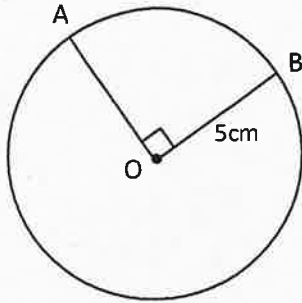
7. Some plastic spherical ball bearings are melted down and remoulded to make a cone.  
The ball bearings have a diameter of 1.6cm.  
The cone must have a radius of 3.4cm and a height of 4.2cm.  
How many ball bearings need to be melted down to make the cone?  
Explain your answer.



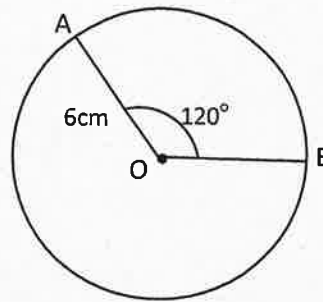


8. Calculate the length of the minor arc AB in each of the following.

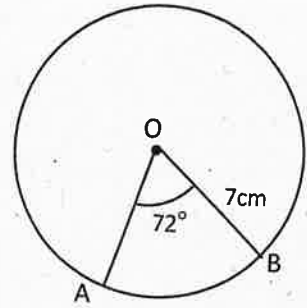
(a)



(b)

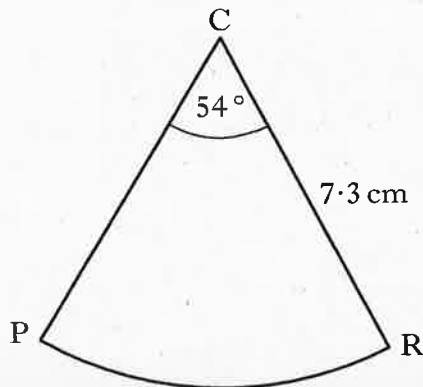


(c)

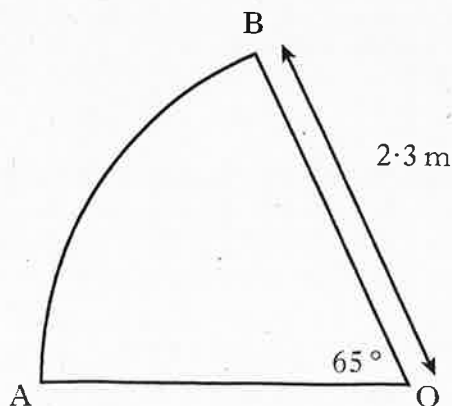


9. Calculate the area of the sector AOB in each of the diagrams in question 8.

10. The diagram shows a sector of a circle, centre C.  
The radius of the circle is 7.3 cm and angle PCR is  $54^\circ$ .  
Calculate the area of the sector PCR.



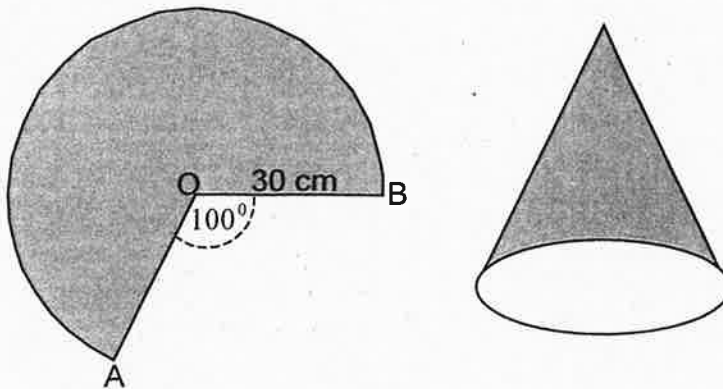
11. A sector of a circle, centre O, is shown below.  
The radius of the circle is 2.3 m.  
Angle AOB is  $65^\circ$ .  
Find the length of the arc AB.



12. A sweet shop sells sweets in cone shaped cardboard containers.

A cone is formed from a cardboard circle with a sector removed as shown in the diagram below.

The radius of the circle is 30cm and angle AOB is  $100^\circ$ .



- Calculate the area of card used to make a cone.
- Calculate the length of the arc AB.
- To strengthen the cone, the shop wants the base of it to be edged in a red plastic trim.  
What is the maximum number of cones which can be edged with 25 metres of plastic trim?

13. Witches hats for Halloween are made from cardboard sectors as shown below.

The radius of OB is 8cm. Angle AOB is  $135^\circ$ .

Black ribbon is used to edge the base of the hats.

What is the maximum number of hats which can be decorated with 125 metres of ribbon?



# ANSWERS

## ASSESSMENT STANDARD 1.1

1. (a)  $2\sqrt{3}$       (b)  $3\sqrt{3}$       (c)  $4\sqrt{2}$       (d)  $5\sqrt{3}$       (e)  $2\sqrt{5}$

(f)  $3\sqrt{5}$       (g)  $2\sqrt{11}$       (h)  $10\sqrt{3}$       (i)  $5\sqrt{5}$       (j)  $6\sqrt{2}$

2. (a)  $y^7$       (b)  $12m^5$       (c)  $g^4$       (d)  $h^6$       (e)  $x^3$

(f)  $3x^3$       (g)  $h^2$       (h)  $y^{-1}$       (i)  $8m^{5/2}$       (j)  $8m^9$

3. (a)  $1.55 \times 10^8$  seconds

(b)  $\text{£}2.8 \times 10^6$

(c)  $1.8414 \times 10^{-2}$  kg

## ASSESSMENT STANDARD 1.2

1. (a)  $5x + 20$  (b)  $8x - 12y$  (c)  $3a^2 + ab$   
(d)  $5m^2 - 6m$  (e)  $2y^2 - 6y$  (f)  $2y^2 + xy$   
(g)  $x^2 + 8x + 15$  (h)  $m^2 + m - 12$  (i)  $h^2 - 8h + 15$   
(j)  $x^2 + x - 20$  (k)  $x^2 + 12x + 36$  (l)  $x^2 - 10x + 25$

2.  $x^2 + 6x + 5$

3. Right rectangle =  $x^2 + 12x + 27$

Left rectangle =  $x^2 + 4x - 5$

Total =  $2x^2 + 16x + 22$

4. Large rectangle =  $x^2 + 11x + 24$

Small rectangle = 12

Shaded area =  $x^2 + 11x + 12$

5. (a)  $2(x + 3)$  (b)  $7(2y - 1)$  (c)  $2(2a + 3b)$   
(d)  $5(2x - 5y)$  (e)  $y(y + 7)$  (f)  $a(a - 3)$   
(g)  $d(3d + 5)$  (h)  $3m(2m - 1)$  (i)  $(x + 4)(x - 4)$   
(j)  $(y + 5)(y - 5)$  (k)  $(a + 10)(a - 10)$  (l)  $(p + 1)(p - 1)$   
(m)  $(x + 5)(x + 1)$  (n)  $(y + 3)(y + 2)$  (o)  $(h + 5)(h + 3)$   
(p)  $(x + 6)(x + 2)$

6. (a)  $(x + 3)^2 - 5$  (b)  $(x + 4)^2 - 15$  (c)  $(x + 4)^2 - 26$   
(d)  $(x + 2)^2 - 3$  (e)  $(x + 5)^2 - 33$  (f)  $(x + 1)^2 - 4$

ASSESSMENT STANDARD 1.3

1. (a)  $x + 4$

(b)  $\frac{1}{x-5}$

(c)  $\frac{(x-6)^2}{x+4}$

(d)  $\frac{x+6}{x-7}$

(e)  $\frac{3x+5}{2x+1}$

(f)  $\frac{2x+1}{4x-7}$

2. (a)  $\frac{49}{40}$

(b)  $\frac{6y+3x}{xy}$

(c)  $\frac{4b+3a}{ab}$

(d)  $\frac{8y-3x}{xy}$

(e)  $\frac{2n-3m}{mn}$

(f)  $\frac{15}{28}$

(g)  $\frac{4}{7}$

(h)  $\frac{12}{xy}$

(i)  $\frac{4b}{ac}$

(j)  $\frac{5}{4}$

(k)  $\frac{bc}{4d}$

(l)  $\frac{xz}{5y}$

## ASSESSMENT STANDARD 1.4

1. (a)  $\frac{1}{2}$  (b) 2 (c) Line CD is steeper. It has the higher gradient.

2. (a) 1 (b)  $\frac{1}{5}$  (c) Line EF is steeper. It has the higher gradient.

3.  $\frac{12}{-5}$

4.  $\frac{-4}{3}$

5. Cylinder =  $504.61 \text{ cm}^3$  Cone =  $417.49 \text{ cm}^3$  Sphere =  $16201.95 \text{ cm}^3$

6. Volume of tank =  $2923.92 \text{ cm}^3$  Volume of tub =  $2815.64 \text{ cm}^3$

The tub has a smaller volume than the tank so it will overflow.

7. Volume of each sphere =  $2.14 \text{ cm}^3$  Volume of cone =  $50.82 \text{ cm}^3$

Number of ball bearings needed =  $50.82 \div 2.14 = 23.75$

24 ball bearings are needed.

8. (a) 7.85 cm (b) 12.56 cm (c) 8.792 cm

9. (a)  $19.63 \text{ cm}^2$  (b)  $37.68 \text{ cm}^2$  (c)  $30.77 \text{ cm}^2$

10.  $25.10 \text{ cm}^2$

11. 2.61 m

12. (a)  $2041 \text{ cm}^2$  (b) 136.07 cm (c) 18 cones

13. (a) 663 hats