EXAM HOMEWORK 5

2013

10. a) $|z+i| = 1$

$|x+iy+i| = 1$

$|x+i(y+1)| = 1$

$x^2 + (y+1)^2 = 1^2$

Circle centre $(0,-1)$ ✓ radius 1 ✓

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b) $|z-1| = |z+5|$

$|x+iy-1| = |x+iy+5|$

$|(x-1)+iy| = |(x+5)+iy|$ ✓

$(x-1)^2 + y^2 = (x+5)^2 + y^2$

$x^2 - 2x + 1 = x^2 + 10x + 25$

$-12x = 24$

$x = -2$ ✓

Vertical line $x = -2$ ✓ 3

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A7. Prove $4^n - 1$ is divisible by 3 $\forall n \in \mathbb{Z}^+$

For $n=1$; $4^n - 1 = 4^1 - 1$

$= 4 - 1$

$= 3$ which is divisible by 3. ∴ Result is true for $n=1$. ✓

Assume true for $n=k$; $4^k - 1 = 3p$ * $p \in \mathbb{Z}^+$ ✓

Consider $n = k+1$; LHS $= 4^{k+1} - 1$ ✓

$= 4 \cdot 4^k - 1$

$= 4(3p+1) - 1$ from *

$= 12p + 4 - 1$

$= 12p + 3$

$= 3(4p+1)$ which is divisible by 3 as $4p+1 \in \mathbb{Z}^+$. ✓

The result is true for $n=1$ and if true for $n=k$ it is also true for $n=k+1$. Hence, by induction, it is true $\forall n \in \mathbb{Z}^+$. ✓

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