EXAM HOMEWORK 8

2015

5. $A = \begin{pmatrix} p & 2 & 0 \\ 3 & p & 1 \\ 0 & -1 & -1 \end{pmatrix}$

$\det A = p\begin{vmatrix} p & 1 \\ -1 & -1 \end{vmatrix} - 2\begin{vmatrix} 3 & 1 \\ 0 & -1 \end{vmatrix} + 0$ ✓

$= p(-p+1) - 2(-3-0)$

$= -p^2 + p + 6$

Singular $\Rightarrow \det A = 0$ ✓

$\therefore p^2 - p - 6 = 0$ ✓

$(p+2)(p-3) = 0$

$p = -2, p = 3$ ✓ 4

2014

5. $\underline{u} = 5\underline{i} + 13\underline{j}$ $\underline{v} = 2\underline{i} + \underline{j} + 3\underline{k}$ $\underline{w} = \underline{i} + 4\underline{j} - \underline{k}$

$= \begin{pmatrix} 5 \\ 13 \\ 0 \end{pmatrix}$ $= \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ $= \begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix}$

$\underline{u} \cdot (\underline{v} \times \underline{w}) = \begin{vmatrix} 5 & 13 & 0 \\ 2 & 1 & 3 \\ 1 & 4 & -1 \end{vmatrix}$ ✓

$= 5\begin{vmatrix} 1 & 3 \\ 4 & -1 \end{vmatrix} - 13\begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix} + 0$ ✓

$= 5(-1-12) - 13(-2-3)$

$= -65 + 65$

$= \underline{0}$ ✓ 3

$\underline{u}$ is perpendicular to $\underline{u} \times \underline{v}$. ✓

$\therefore \underline{u}, \underline{v}$ and $\underline{w}$ lie in the same plane. 1

2015

18. a) $\frac{dV}{dt} = -k\sqrt{h}$ $\quad V = Ah$

$\frac{dV}{dh} = A \therefore \frac{dh}{dV} = \frac{1}{A}$

$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$ ✓

$= \frac{1}{A} \times -k\sqrt{h}$ ✓

$= -\frac{k}{A}\sqrt{h}$

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b) $\frac{dh}{dt} = -\frac{k}{A}\sqrt{h}$

$\int \frac{1}{\sqrt{h}}\,dh = -\frac{k}{A}\int 1\,dt$ ✓

$\int h^{-\frac{1}{2}}\,dh = -\frac{k}{A}\int 1\,dt$

$2h^{\frac{1}{2}} = -\frac{k}{A}t + c$ ✓

$\sqrt{h} = -\frac{k}{2A}t + c$

$\sqrt{144} = -\frac{k}{2A}(0) + c$

$c = 12$

$\therefore \sqrt{h} = -\frac{k}{80k}t + 12$

$\sqrt{h} = -\frac{1}{80}t + 12$

$h = (12 - \frac{1}{80}t)^2$ ✓

At $h = 144$, $\frac{dh}{dt} = -0.3$, $t = 0$

$\frac{dh}{dt} = -\frac{k}{A}\sqrt{h}$

$-0.3 = -\frac{k}{A}\sqrt{144}$ ✓

$0.3A = 12k$

$A = 40k \quad k = \frac{A}{40}$

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c) Empty when $h = 0$ $\therefore (12 - \frac{1}{80}t)^2 = 0$ ✓

$12 - \frac{1}{80}t = 0$

$t = 12 \times 80$

$= 960$ hours

$= 40$ days ✓

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d) $r = 20$ cm $\quad A = 400\pi$

4 days = 96 hours

$k = \frac{A}{40}$

$= \frac{400\pi}{40}$

At $t = 96$, $h = (12 - \frac{1}{80} \cdot 96)^2$

$= (10.8)^2 \Rightarrow \sqrt{h} = 10.8$ ✓

$= 116.64$

$\therefore \frac{dV}{dt} = -k\sqrt{h}$

$= -\frac{400\pi}{40} \cdot 10.8$ ✓

$= -108\pi$

$= -339.292$

$\therefore$ Water delivered to the vegetation at ✓ a rate of $339\text{ cm}^3/\text{hr}$ (3sf) (or $108\pi\text{ cm}^3/\text{hr}$)

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